

Optimal Negative Interest on Reserves^a

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Abstract

A negative interest rate on reserves is effectively a tax on the banking system. The rationale for adopting such policy in the context of a classic liquidity trap is its expansionary effect, via the reduction in the incentive to save. However, critics argue that the resulting distortion in the banking system may have adverse effects which outweigh the welfare gains from economic stimulus. This paper contributes to the debate by modelling the banking sector in accordance with the maturity-transformation framework of [Diamond and Dybvig \(1983\)](#). I show that, on the one hand, negative interest on reserves leads to financial disintermediation, which reduces the extent of maturity transformation below the first-best level; while, on the other hand, it is expansionary, as consumers save less overall. The paper's main finding is that, while in normal times taxing banks is inefficient, optimal monetary policy in the liquidity trap prescribes a strictly negative interest on reserves.

Keywords: Interest on reserves, lower bound, maturity transformation, unconventional monetary policies.

JEL Codes: E43, E52, E58, G21.

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1 Introduction

In the aftermath of the Great Recession, the interest rate on bank reserves has become the active monetary policy tool in advanced economies. Central banks increased the supply of reserves massively. This turned the monetary system into a floor system, in which the stance of monetary policy is determined by the interest rate on reserves rather than by the scarcity of bank reserves (i.e., open market operations).¹ Also, the implementation of negative interest rate policies, performed by several advanced-economy central banks, required the active use of the interest on reserves.²

In light of this development in the practice of monetary policy, in this paper I study the role of the interest on reserves in pursuing the central bank's policy objectives. A literature has recently developed on the role of reserves and of the interest on reserves in monetary policy (Kashyap and Stein, 2012; Ennis, 2014; Reis, 2016). However, such literature abstracts from negative interest rates and the challenges to monetary policy posed by the lower bound on nominal interest rates. A separate literature studies the effectiveness of negative interest rates in pursuing macroeconomic objectives when an economy is in the liquidity trap (Rognlie, 2016; Brunnermeier and Koby, 2017). However, as most of the literature on negative interest rates, their analysis concentrates on only one interest rate, the interbank rate. This simplification is more restrictive than usual when negative interest rates are the subject, because it does not allow the analysis to focus on the key issue of the transmission of monetary policy to the money market. I provide a theoretical contribution to the literature on negative interest rates, which I expand by including insights from the literature on reserves.

In the model, I explicitly model monetary policy as a choice of interest on reserves and open market operations, which determines the interbank rate. This a more realistic description of the economy and of monetary policy. Central banks influence multiple interest rates. This explicit modelling of monetary policy transmission allows me to study whether the lower bound on nominal interest rates constrains different interest rates in different ways, and if this has implications for monetary policy in the liquidity trap. The contribution of my paper moves the current debate on negative interest rates closer to the original literature on the liquidity trap, as developed from the seminal paper by Krugman (1998), which is founded on the idea that the interbank rate cannot fall below a lower bound.

The lower bound on the interbank rate represents the idea that a low enough interest on reserves will fail to ease monetary policy via the conventional channel, by lowering the interbank rate and thus, for example, reducing the rewards from saving. In policy circles the question is not whether there is a lower bound, but how to estimate it (Alsterlind et al., 2015; Bech and Malkhozov, 2016; Dell'Ariccia et al., 2017). A recently disclosed Federal Reserve internal memo (Burke et al., 2010) estimates that the federal funds rate can at

¹For example, the Federal Reserve in the last quarter of 2008 alone issued \$800bn in bank reserves, amounting to a nine-fold increase in the supply of reserves. It did not have the authority to pay interest on bank reserves. So, the Emergency Economic Stabilization Act of 2008 granted it such authority on 1 October 2008. This allowed the central bank to continue controlling the fed funds rate, as banks became satiated in reserves.

²Please refer to figure 1 in appendix A for data on the implementation of negative interest on reserves.

most be lowered to -35 basis points.³ Considering that the current interbank rate in the Euro Area is -0.35%, hitting the lower bound is a real possibility in case further monetary easing were desirable.⁴ Therefore, a full evaluation of the desirability of negative interest rate policies must take this eventuality into account.⁵

The interest on reserves is not constrained by a lower bound in the same way as the interbank rate is. A first-pass explanation for this is that the central bank can decide by decree to pay whatever interest on bank reserves, while the interbank rate is the equilibrium price that clears the interbank market. The central bank controls the interbank rate only indirectly by carrying out open market operations and by setting the interest on reserves, and I show that the lower bound emerges as an equilibrium requirement due to the existence of zero-interest-paying physical currency.⁶

Nonetheless, this does not imply that the central bank can ease monetary policy just by lowering the interest on reserves, once the interbank rate is stuck at the the lower bound. Consider the canonical New Keynesian model, where the banking sector is just a veil behind which consumers lend to firms. In such model, lowering the interest on reserves per se would not be expansionary, because, even if banks were forced to hold bank reserves, consumers would move all of their savings out of deposits into the money market if the cut in the interest on reserves led to a reduction in the deposit rate. In other words, in such framework deposits and bonds are perfect substitutes from the consumers' perspective and therefore the interest on reserves has a macroeconomic role only insofar as it steers the interest rate in the bond market. In order to meaningfully study the macroeconomic effects of the interest on reserves, we need a theory of banking which generates a demand for deposits.

The role of banks in the model that I develop is based on the classical framework by [Diamond and Dybvig \(1983\)](#) on maturity transformation. Banks are depository institutions. Consumers are subject to idiosyncratic liquidity shocks, which may force them to wind up their investments early and thus forego the liquidity premium on their assets. The combination of liquidity shocks and liquidity premium gives rise to liquidity risk for consumers. Banks emerge in the model to provide insurance against liquidity risk. They do so by issuing redeemable deposits and holding long-term assets. In other words, banks perform maturity transformation, which means that even short-term depositors enjoy to some extent the higher yields on the long-term assets that are held by their bank. The key role of banks in this model is to facilitate saving behaviour, as they provide consumers with assets that are more liquid than the assets available in direct asset markets. This also implies that deposits and direct holdings of assets are not perfect substitutes from the perspective of consumers.

An influential strand of the literature on negative interest rates also has its focus

³[Burke et al. \(2010\)](#) is the only publicly accessible paper that reports the methodology with which the lower bound on the interbank rate is estimated. [Viñals et al. \(2016\)](#) reports estimates by IMF staff ranging from -75 basis points to -200 basis points, but without disclosing the analysis.

⁴The Eonia rate is the overnight interbank rate in the Euro Area. It is -0.35% as of 6 November 2017. Concurrently, the interest on excess reserves in the Euro Area is -0.4%.

⁵[Bernanke \(2016\)](#) makes this point to argue that negative interest rate policies are unlikely to have large stimulative effects.

⁶[Hicks \(1937\)](#) was the first to describe the lower bound on the money-market interest rate in these terms.

on the banking system, but uses a different theory of banking. It studies the interaction of a negative interbank rate with the banking system, by emphasising the asset side of banks: the key role of the banking system in their model is to invest, because banks are defined by their high productivity at making investments. They show that, if deposit rates are downwardly sticky at zero, there is a level of the interbank rate beyond which cuts are contractionary, because they harm bank profitability and thus investment (Brunnermeier and Koby, 2017; Eggertsson et al., 2017). The framework that I propose is not in contradiction with this strand of the literature. In fact, it complements it by focusing on another one of the many different roles that banks play in the economy: maturity transformation.

Since in my model deposits and direct holdings of assets are not perfect substitutes, I find that a reduction in the interest on reserves does not only lead to disintermediation. Interestingly, it also leads to a reduction in savings and thus to an increase in aggregate demand. The mechanism whereby the interest on reserves affects consumers' propensity to save runs through the deposit rate. Banks hold reserves, either because they are required to do so or because reserves are useful in the payment system among banks. Thus, a reduction in the interest on reserves translates into a lower deposit rate. The reduction in the deposit rate makes the assets available to consumers less attractive overall, because deposits and other assets are imperfectly substitutable. Hence, consumers respond both by moving their wealth out of deposits into direct holdings of assets and by saving less in total.

A benevolent central bank finds that boosting aggregate demand by reducing the interest on reserves, while holding the interbank rate constant, is beneficial, if aggregate demand is insufficient because the interbank rate is at its lower bound. However, this policy has a drawback: it reduces the level of bank activity. In this economy banks provide liquidity-risk insurance, a desirable service. Therefore, an interesting policy tradeoff emerges between stimulating the economy and preserving the banking system.

The main finding of the paper is that, when the lower bound on the interbank rate is binding and the economy needs further easing, it is optimal to lower the interest on reserves strictly below the interbank rate. This boosts employment but also reduces the effectiveness of the banking system. The key is that the point at which the negative effect of the latter outweighs the benefits of the former is strictly below the lower bound. When the economy is in the liquidity trap, the central bank should exploit this channel of monetary policy in order to stimulate the economy by boosting demand. The result is important for monetary policy in the liquidity trap, because it identifies an unconventional policy instrument that can improve the performance of the economy when further reductions in the interbank rate cannot be implemented.

Rogoff (2017) identifies the lower bound on the conventional monetary policy instrument (i.e., the interbank rate) as the key constraint currently facing central bankers. Moreover, there is a growing literature (Karabarbounis and Neiman, 2013; Summers, 2014; Carvalho et al., 2016) arguing that structural factors are pushing down the real interest rate, resulting in a higher probability of hitting the lower bound in the future. This suggests that the literature on unconventional monetary policy, in which this paper fits, is relevant and will remain relevant in the future.

1.1 Related literature

Unconventional monetary policies, defined as policies within the remit of the central bank other than targeting the spot short-term interbank interest rate, have attracted a large academic literature.

This paper is closely related to a small number of papers that study negative interest rate policies. [Rognlie \(2016\)](#) postulates a demand for currency with a bliss point within a New Keynesian set-up and finds that transmitting a negative interest rate to the bonds market is feasible. Moreover, he finds that it is desirable when the economy experiences a liquidity trap. [Brunnermeier and Koby \(2017\)](#) and [Eggertsson et al. \(2017\)](#) study the interaction between negative rates and banks. They assume a lower bound on deposit rates and find that negative interest rate policies reduces banks' interest margin and profits. Since banks are subject to a capital constraint and are the only lenders in the economy, negative interest rate policies, which lower bank profits, are contractionary. The evidence for a lower bound on deposit rates is mixed. [Bech and Malkhozov \(2016\)](#) and [Heider et al. \(2018\)](#) find evidence for it. The latter show that retail banks in the Eurozone more reliant on deposits suffered more from the European Central Bank's negative interest rate policy. On the other side of the argument, [Basten and Mariathan \(2017\)](#) show that Swiss retail banks funded with more deposits, which should have been hit harder by negative interest rate policies in the presence of a binding zero lower bound on deposit rates, increased fees on depositors more and hence did not suffer in terms of profits. Effectively, this means that negative interest rate policies were passed on to depositors. In my paper, I model the banking sector in a novel way in the context of this literature, as a maturity transformer, and I focus on the lower bound on the interbank rate, which has a long tradition in the New Keynesian liquidity trap and can be derived from the presence of paper money.

A theory of banking based on the notion of maturity transformation was first formalised by [Diamond and Dybvig \(1983\)](#). A subsequent theoretical literature worked on expanding the framework with general-equilibrium elements. [Jacklin \(1987\)](#) and [Allen and Gale \(2004\)](#) add financial markets and discuss the implications for the incentive compatibility of the demand deposit contract.⁷ [Hellwig \(1994\)](#) and [Farhi et al. \(2009\)](#) introduce competitive forces in the banking sector.

The existence of a lower bound on the nominal interest rate prevailing in the bonds market, which acts as a constraint on monetary policy, was first hypothesised by [Hicks \(1937\)](#). The seminal paper for the modern literature is [Krugman \(1998\)](#), and a very large literature followed. An integration of the lower-bound friction in the canonical New Keynesian framework, as laid out by [Woodford \(2003\)](#) and [Galí \(2008\)](#), is reached in [Eggertsson and Woodford \(2003\)](#).⁸ Demand shocks are commonly modelled as simple time-preference shocks within the literature. More structure on the large adverse demand shock that leads into the liquidity trap is provided by [Guerrieri and Lorenzoni \(2011\)](#) and [Eggertsson and Krugman \(2012\)](#), who model the adverse demand shock as a consequence of a credit crisis.

This paper finds that the interest rate on reserves has macroeconomic effects per se,

⁷See also [Diamond \(1997\)](#) and [von Thadden \(1998\)](#) on this subject. A discussion of the incentive compatibility of maturity transformation with multiple assets is available in [von Thadden \(1997\)](#).

⁸In [Werning \(2011\)](#) a similar model is presented in continuous time.

beyond the mere transmission of the monetary policy stance to the interbank rate. Bank reserves and the interest rate on reserves recently received a great deal of attention. [Reis \(2016\)](#) discusses monetary policy by central banks with large balance sheets. He finds that maintaining banks satiated in their reserve holdings is desirable, because it makes the choice of interest rate independent of the balance-sheet size. This frees quantitative easing for use as an independent policy instrument that can achieve independent policy goals. [Cúrdia and Woodford \(2011\)](#) also advocate a floor system of monetary policy on the ground that a system which sets the interest rate on reserves below the interbank rate is effectively taxing the banking sector. Eliminating such tax implements the Friedman rule and is therefore optimal. Moreover, they also find that the floor system has the benefit of making the central bank's balance-sheet size independent of the target interest rate. Thus, it can be used to react to financial-intermediation disturbances. [Kashyap and Stein \(2012\)](#) contrast these views by pointing out that, if short-term debt issued by the banking sector poses a systemic risk, it is optimal to set a non-zero wedge between the interbank rate and the interest rate on reserves in order to pursue macroprudential objectives. The interest-rate wedge serves as a tax, disincentivising excessive issuance of short-term debt by the banking sector. This paper's finding that optimal monetary policy in the liquidity trap requires a positive wedge between the interbank rate and the interest on reserves is similar: the interest rate wedge acts as a tax on bank deposits, discouraging excessive saving by consumers.

2 Technology and Preferences

This section lays out the assumptions of the model on technology and preferences. They are mostly similar to the assumptions made in [Diamond and Dybvig \(1983\)](#), the seminal paper in the literature on maturity transformation, with differences that I will highlight.

There is an investment technology, which transforms consumption goods into capital goods, according to function

$$K = f(I), \tag{1}$$

with f twice-continuously differentiable, $f' > 0$ and $f'' < 0$. This assumption is different from [Diamond and Dybvig \(1983\)](#), where $f' = 1$ for all I . I need this new assumption to have an endogenous real interest rate in the model.

Each unit of the capital good returns one unit of the consumption good if liquidated after one period, and $R > 1$ units of the consumption good if liquidated after two periods. Hence, there is a reward from holding the asset longer. I call this the liquidity premium.

The economy is inhabited by a unit mass of ex-ante identical consumers indexed by j . Consumers live for three periods. At time zero, they receive an endowment of consumption goods and make a saving decision. In the subsequent time periods, consumers consume the proceeds of their time-0 savings.

Consumers' preferences are represented by utility function

$$U(C_{j,0}, C_{j,1}, C_{j,2}, \theta_j) = u(C_{j,0}) + \hat{\beta} \cdot [(1 - \theta_j) \cdot u(C_{j,1}) + \beta \cdot \theta_j \cdot u(C_{j,2})], \tag{2}$$

with felicity function u satisfying Inada conditions. $\beta \in [R^{-1}, 1]$ is the discount factor between time 1 and time 2. $\hat{\beta}$ is the discount factor between time 0 and the future. Shocks to $\hat{\beta}$ represent demand shocks at time 0. A necessary assumption is that the coefficient of relative risk aversion is greater than 1:

$$\frac{-C \cdot u''(C)}{u'(C)} \equiv \gamma(C) \geq 1 \quad \forall C > 0. \quad (3)$$

Risk aversion needs to be high enough to generate the demand for liquidity-risk insurance that is at the heart of this paper. The utility function is different than in [Diamond and Dybvig \(1983\)](#) in that consumers enjoy consumption also at time 0. To analyse interest-rate setting, the model needs a meaningful saving decision.

Random variable θ_j represents a liquidity shock and, as such, it is only known at time 1. It takes on values 0 or 1. At time 0, agents know the objective probability of the liquidity shock's realisations:

$$Pr(\theta_j) = \begin{cases} \phi & \text{if } \theta_j = 0, \\ 1 - \phi & \text{if } \theta_j = 1. \end{cases} \quad (4)$$

A consumer whose realisation for θ_j is 0 is hit by the liquidity shock. I refer to these consumers throughout the paper as early types and to the other consumers, who have not been hit by the liquidity shock, as late types. There is no uncertainty at time 0 about the share of consumers who will be hit by the liquidity shock. Hence, there is no aggregate risk.

3 Social Planner

The allocation of the social planner is the benchmark for efficiency in the following analysis of the decentralised economy.

The social planner maximises the expected value of aggregate welfare subject to the economy's resource constraints and non-negativity of consumption. Informational frictions, which are a key characteristic of the decentralised economy, do not constrain the social planner. It follows that the social planner represents a high standard of efficiency.

Since the social planner finds it optimal to let consumers of the same type θ_j consume equally (i.e., $C_{j,t}(\theta_j) = C_{h,t}(\theta_j) = C_t(\theta_j)$), expected aggregate welfare can be written as

$$u(C_0) + \hat{\beta} \cdot \{\phi \cdot u[C_1(0)] + (1 - \phi) \cdot \beta \cdot u[C_2(1)]\}. \quad (5)$$

The economy's resource constraints are given by:

$$C_0 + I \leq Y, \quad (6)$$

$$\phi \cdot C_1(0) + (1 - \phi) \cdot C_1(1) \leq L, \quad (7)$$

$$\phi \cdot C_2(0) + (1 - \phi) \cdot C_2(1) \leq R \cdot [f(I) - L], \quad (8)$$

where L stands for the quantity of capital goods liquidated at time 1.

Consumption must be non-negative at all points in time and for all consumers, as according to

$$C_t(\theta_j) \geq 0 \quad \forall t, \theta_j. \quad (9)$$

The allocation that solves the social planner's problem is by definition first-best efficient. The system of equations that pins down the efficient allocation of consumption across types and time $\{C_0, C_1(\theta_j), C_2(\theta_j)\}_{\theta_j \in \{0,1\}}$ is given by:

$$u'(C_0) = f'(Y - C_0) \cdot \hat{\beta} \cdot u'[C_1(0)], \quad (10)$$

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = R, \quad (11)$$

$$(1 - \phi) \cdot C_2(1) = R \cdot [f(Y - C_0) - \phi \cdot C_1(0)], \quad (12)$$

$$C_1(1) = C_2(0) = 0. \quad (13)$$

Definition 1.

An allocation $\{C_0, C_1(\theta_j), C_2(\theta_j)\}_{\theta_j \in \{0,1\}}$ is first-best efficient if it satisfies equations (10), (11), (12) and (13).

Equation (10) is an Euler equation. The optimal saving decision at time zero sets the marginal rate of substitution between time 0 and time 1 equal to the corresponding marginal rate of transformation.

Equation (11) is familiar from the optimality conditions in the standard Diamond-Dybvig model. It implies that it is optimal to insure liquidity risk in the economy. If consumers do not insure each other, then early types, who are forced to liquidate their asset early, consume too little. Ex ante, in a fully efficient economy the provision of liquidity-risk insurance implements equation (11).

4 Decentralised Economy

In this section, I describe the characteristics of the four types of agents who inhabit the decentralised economy: capital-producing firms, consumers, banks and the government. I

4.1 Capital-Producing Firms

Capital-producing firms purchase consumption goods and transform them into capital goods, which they sell. They are active at time 0 within a perfectly competitive market for capital goods. A representative firm statically maximises its profits Π_F , given by

$$\Pi_F = Q \cdot K - I, \quad (14)$$

where Q is the price of capital in terms of consumption goods, I is the quantity of consumption goods purchased by the firm and invested to produce K units of capital goods. They produce capital goods according to technology

$$K = f(I), \quad (1)$$

with f twice-continuously differentiable, $f' > 0$ and $f'' < 0$.

As described above, capital production implies an upward-sloping supply of capital goods. The Diamond-Dybvig model is a limiting case, where capital supply is perfectly elastic at $Q = 1$. The underlying technological assumption of [Diamond and Dybvig \(1983\)](#) is that $f'(I) = 1$ and $f''(I) = 0$ for all I . Decreasing returns to investment, as assumed in this paper, are necessary to make the interest rate endogenous.

4.2 Consumers

Consumers make saving and portfolio allocation decisions under idiosyncratic liquidity risk. There is a unit measure of consumers, who are identical as of time 0. Consumer j 's utility function is given by

$$U(C_{j,0}, C_{j,1}, C_{j,2}, \theta_j) = u(C_{j,0}) + \hat{\beta} \cdot [(1 - \theta_j) \cdot u(C_{j,1}) + \beta \cdot \theta_j \cdot u(C_{j,2})]. \quad (2)$$

All consumers enjoy consumption at time 0. Then, at time 1 they are subject to a privately-observed liquidity shock θ_j : with probability $\phi \in (0, 1)$ they become early types, with $\theta_j = 0$, and enjoy consumption only at time 1; with probability $1 - \phi$ they become late types, with $\theta_j = 1$, and enjoy consumption only at time 2. The consumer's expected utility function is therefore

$$U[C_{j,0}, C_{j,1}(0), C_{j,2}(1)] = u(C_{j,0}) + \hat{\beta} \cdot \{\phi \cdot u[C_{j,1}(0)] + (1 - \phi) \cdot \beta \cdot u[C_{j,2}(1)]\}. \quad (15)$$

Consumers are subject to budget constraints. At time 0, they receive an endowment Y , transfers from the government T and profits from banks Π_B and firms Π_F . They use this income to consume and save. Savings are held in bank deposits $\int_0^1 D_{j,k} dk$, where k is an index that represents a bank, or invested directly in capital goods. So, the time-0 budget constraint is given by

$$C_{j,0} + Q_0 \cdot K_j + \int_0^1 D_{j,k} dk = Y + T + \Pi_B + \Pi_F. \quad (16)$$

The ability of the consumer to invest directly in capital goods at time 0 is key in the model. It makes the demand for financial intermediation endogenous in the model. If deposits become less attractive, consumers have the possibility to change the portfolio allocation of their savings tilting it more towards direct finance.

At time 1, the idiosyncratic liquidity shock is realised. Thus, consumer's decisions at time 1 are contingent on their type realisation. To finance consumption $C_{j,1}(\theta_j)$, they can liquidate $L_j(\theta_j)$ units of the physical capital or withdraw $\int_0^1 W_{j,k} dk$ from their bank deposit. Consumer j 's budget constraint at time 1 is then given by

$$C_{j,1}(\theta_j) = L_j(\theta_j) + \int_0^1 W_{j,k}(\theta_j) dk. \quad (17)$$

I do not allow consumers to sell their capital goods at time 1 or invest in capital goods at time 1 at all. Since it removes any incentive for late-type consumers to withdraw their

deposits early, this assumption eliminates the incentive-compatibility constraint from the bank's problem and greatly simplifies the model. A restrictive assumption on the time-1 market for used capital is necessary to escape the Jacklin critique (Jacklin, 1987) and retain maturity-transforming banks. According to the Jacklin critique, if consumers are allowed to frictionlessly borrow and lend at time 1 (or buy and sell used capital), then the banking mechanism is unable to provide any liquidity-risk insurance. For example, Diamond and Dybvig (1983) has maturity transformation in equilibrium, because consumers are implicitly not allowed to lend and borrow among themselves at time 1. They can only reinvest in new capital goods at time 1. In this paper, I make a more extreme assumption. However, I think this is justified on the grounds that my focus is not the fragility of the banking contract, for which late-type consumers' desire to withdraw early is key, but the demand for deposits due to banks' maturity-transformation activity.

At time 2, consumers use their remaining assets to purchase consumption goods $C_{j,2}(\theta_j)$ according to

$$C_{j,2}(\theta_j) = R \cdot [K_j - L_j(\theta_j)] + \int_0^1 (1 + d_{k,1}) \cdot [(1 + d_{k,0}) \cdot D_{j,k} - W_{j,k}(\theta_j)] dk. \quad (18)$$

$d_{k,0}$ and $d_{k,1}$ are the real interest rates, respectively from time 0 to time 1 and from time 1 to time 2, offered in the deposit contract of bank k .

Before I present the bank's problem, it is useful to work out consumers' optimal decisions with regard to withdrawing and depositing. The consumer's withdrawing behaviour is simple, thanks to the assumption that there is no market for loans and capital goods at time 1. Early types withdraw early and late types withdraw late, as formalised by

$$W_{j,k}(\theta_j) = \begin{cases} (1 + d_{k,0}) \cdot D_{j,k} & \text{if } \theta_j = 0, \\ 0 & \text{if } \theta_j = 1. \end{cases} \quad (19)$$

Consumer j 's demand for bank k 's deposits depends on the return and liquidity-insurance characteristics that the bank offers relative to other deposit contracts on offer. It is a Bertrand-like demand function, where the consumer deposits everything with the bank that offers the best contract from the consumer's viewpoint. From the consumer's problem, I derive a valuation function for deposit contracts according to which the consumer chooses her bank. It determines the value of a unit of bank k 's deposits in utility terms for consumer j :

$$V_j(d_{k,0}, d_{k,1}) \equiv (1 + d_{k,0}) \cdot \hat{\beta} \cdot \{\phi \cdot u'[C_{j,1}(0)] + (1 - \phi) \cdot (1 + d_{k,1}) \cdot \beta \cdot u'[C_{j,2}(1)]\}. \quad (20)$$

Demand for bank k 's deposits is given by

$$D_{j,k} = \begin{cases} D_j & \text{if } V_j(d_{k,0}, d_{k,1}) > \max_{z \neq k} \{V_j(d_{z,0}, d_{z,1})\} \\ [0, D_j] & \text{if } V_j(d_{k,0}, d_{k,1}) = \max_{z \neq k} \{V_j(d_{z,0}, d_{z,1})\} \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

$D_j \equiv \int_0^1 D_{j,k} dk$ is defined as the total amount of deposits held by consumer j . It is determined in equilibrium. The consumer decides to hold positive amount of deposits if the

value of the best deposit contract, $\max_k\{V_j(d_{k,0}, d_{k,1})\}$, is at least as good as the value of investing directly in capital goods, given by

$$V_{j,K} \equiv Q \cdot \hat{\beta} \cdot \{\phi \cdot u'[C_{j,1}(0)] + (1 - \phi) \cdot R \cdot \beta \cdot u'[C_{j,2}(1)]\}. \quad (22)$$

4.3 Banks

There is a unit mass of banks in a perfectly competitive market. They are profit-maximising agents that supply deposits D_k , characterised by interest rates $(d_{k,0}, d_{k,1})$, and use the resources they collect to invest in capital assets K_k , bank reserves B_k , currency Cu_k and interbank loans F_k . The latter three assets are nominal. Bank k 's profits are defined as

$$\Pi_k + Q \cdot K_k + \frac{B_k + Cu_k + F_k}{P_0} = D_k. \quad (23)$$

The bank's portfolio allocation decision is constrained by a reserve requirement. Banks must hold at least a proportion $\rho \in [0, 1]$ of their deposits in reserves, according to

$$\frac{B_k}{P_0} \geq \rho \cdot D_k. \quad (24)$$

From equation (19), banks know how many of their deposits will be withdrawn at time 1. They use their short-term asset holdings, reserves, currency and interbank loans, and liquidate capital goods L_k to satisfy early withdrawal requests, as according to

$$\phi \cdot (1 + d_{k,0}) \cdot D_k = L_k + \frac{(1 + i^B) \cdot B_k + Cu_k + (1 + i) \cdot F_k}{P_1}. \quad (25)$$

The nominal interest rates on reserves and interbank loans are respectively given by i^B and i . Of course, currency pays a zero nominal interest rate. This generates a zero lower bound on the interest rate that can be paid on interbank loans. The absence of storage costs is an assumption made for notational convenience. Storage costs that are proportional to currency holdings would lower the effective lower bound to negative territory but would not change the paper's results once such effective lower bound becomes binding. Late deposit withdrawals are met with the bank's remaining assets, according to

$$(1 - \phi) \cdot (1 + d_{k,1}) \cdot (1 + d_{k,0}) \cdot D_k = R \cdot (K_k - L_k). \quad (26)$$

Banks know the consumers' demand for their their deposits $\int_0^1 D_{j,k} dk$, given by equation 21. Accordingly,

$$D_k = \int_0^1 D_{j,k} dj. \quad (27)$$

4.4 Government

The monetary base, M , is supplied by the government at time 0. On the asset side, the government engages in open-market operations, whereby it purchases capital goods K_g . Accordingly, the government's time-0 budget constraint is

$$Q \cdot K_g + T = \frac{M}{P_0}. \quad (28)$$

T are lump-sum transfers to consumers whereby the government rebates its seigniorage revenue.

The split of the monetary base in reserves and currency is not directly determined by the government. It is determined in equilibrium by relative demand for bank reserves and currency. Given that there is no special role for currency in the model, except as an asset that pays a zero percent nominal return, currency is only held in non-zero amount if the interest on reserves i^B is smaller than zero. The government controls directly the rate of interest on reserves i^B . At time 1, the government uses its capital holdings to pay off holders of the monetary base, according to

$$K_g = (1 + i^B) \cdot \rho \int_0^1 D_k dk + (1 + \max\{i^B, 0\}) \cdot \left(\frac{M}{P_1} - \rho \cdot \int_0^1 D_k dk \right). \quad (29)$$

At time 2, the government plays no role in the economy.

Note that throughout the paper I also refer to the government as central bank and monetary authority, because I focus on the monetary prerogatives of government.

5 Equilibrium

In this section of the paper, I define two equilibrium concepts. First, I define the competitive equilibrium with flexible prices. This allows me to find the real interest rate that must prevail for markets to clear: the natural real rate of interest. Second, I define an equilibrium concept with sticky prices. In this equilibrium, I can meaningfully study monetary policy, which has real effects. Demand and supply are not automatically equated by changes in the price level. Hence, it is the central bank's task to manage demand in order to ensure the full employment of the endowment. I will assume that the central bank is benevolent and knows exactly the interest rate that ensures market clearing. However, if monetary policy is constrained by the lower bound, then the equilibrium features rationing in the time-0 goods market.

5.1 Flexible-Price Equilibrium and the Natural Real Interest Rate

The flexible-price equilibrium is a useful benchmark to understand the workings of the model. Moreover, it pins down the natural real rate of interest, the real rate of interest at which the goods market clears.

In the flexible-price equilibrium, firms, banks and consumers solve their optimisation problems and prices adjust to ensure market clearing. The government controls the interest on reserves, i^B , and the interest on the interbank market, i . I formalise the equilibrium concept as follows.

Definition 2. *Given policy $\{i^B, i\}$ with $i^B \leq i$, the flexible-price equilibrium consists of quantities $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j), \Pi_f, K, I, \Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, M, K_g, T\}_{j,k,\theta_j}$ and prices $\{P_0, P_1, Q, d_{k,0}, d_{k,1}\}_{k \in [0,1]}$ such that*

1. *The representative capital-producing firm chooses $\{\Pi_f, K, I\}$ to maximise its profits, Π_f , defined by equation (14) subject to (1).*

2. Consumer j chooses $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j)\}$ to maximise (15) subject to budget constraints (16), (17), (18) and non-negativity constraints

$$C_{j,t}(\theta_j) \geq 0 \quad \forall t, \theta_j. \quad (30)$$

3. Bank k chooses $\{\Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, d_{k,0}, d_{k,1}\}$ to maximise its profits, Π_k , subject to budget constraints (23), (25), (26), the reserve requirement (24), demand for deposits given by equations (21) and (27), and non-negativity constraints

$$(B_k, Cu_k, K_k) \geq 0. \quad (31)$$

4. $\{M, K_g, T\}$ are such that the government's budget constraints (28) and (29) hold.

5. The goods market clears with

$$\int_0^1 C_{j,0} dj + I = Y. \quad (32)$$

6. The market for reserves clears with

$$\int_0^1 B_k + Cu_k dk = M. \quad (33)$$

7. The market for interbank loans clears with

$$\int_0^1 F_k dk = 0. \quad (34)$$

8. The market for capital goods clears with

$$\int_0^1 K_j dj + \int_0^1 K_k dk + K_g = K. \quad (35)$$

From here on, I drop the k index for banks, since banks behave symmetrically. Moreover, I drop the index j for consumers realising that heterogeneity between consumers is uniquely due to different realisations of a consumer's liquidity shock, θ .

First of all, I find the zero lower bound as an equilibrium condition. A policy that sets the interest rate in the interbank market strictly below zero is not compatible with equilibrium, because all banks would have an incentive to borrow in order to hold money in the form of currency. The zero lower bound is given by

$$i \geq 0. \quad (36)$$

A storage cost for currency would create an effective lower bound at a negative level. Although the empirical evidence is that the lower bound is effectively at a negative level, in this paper I adopt a zero lower bound for simplicity. The same mechanism as studied in the paper would operate with a negative effective lower bound.

The interbank rate of interest, i , is important in the economy because, by exploiting arbitrage opportunities, banks make sure that it is equated to the short-term return on capital assets, according to

$$(1 + i) \cdot \frac{P_0}{P_1} = Q^{-1}. \quad (37)$$

In fact, if the real interest rate on interbank loans was lower than the short-term return on capital assets, then every bank would want to borrow in this market and this would violate the market clearing condition. Conversely, if the real interbank interest rate was higher, all banks would want to lend in this market.

It is interesting to study whether the short-term real interest rate can be influenced by monetary policy. As is usual in set-ups without nominal rigidities, I find that the short-term real interest rate is decoupled from monetary policy. It is pinned down by the supply schedule for capital goods and by the market-clearing condition of the goods market (48), as

$$Q^{-1} = f'(Y - C_0). \quad (38)$$

In equilibrium, banks offer a deposit contract that only pays off for early types. Late-type consumers do not get any of their deposits back. This is a direct consequence of the absence of an investment technology for consumers at time 1 and thus of the absence of an incentive-compatibility constraint on the bank. Moreover, given that banks Bertrand-compete to supply deposits, banks make zero profits in equilibrium.

Lemma 1. Define $\tau \equiv \rho \cdot \frac{i - i^B}{1 + i}$. In equilibrium, the prevailing deposit contract $\{d_0, d_1\}$ is given by:

$$1 + d_0 = (1 + i) \cdot \frac{P_0}{P_1} \cdot \frac{1 - \tau}{\phi} \quad (39)$$

$$1 + d_1 = 0 \quad (40)$$

Proof. Please refer to appendix B. □

The absence of incentive-compatibility considerations has the drawback of giving an unrealistic deposit contract, where late-type consumers ex-post receive nothing from their time-0 deposits. However, the upside of this specification is that the role of banks as providers of liquidity-risk insurance becomes very clear. It is worth noting that from an ex-ante perspective, all consumers are willing to deposit part of their wealth in bank deposits because of this insurance role.

Using the consumer's first-order condition with regard to capital goods combined with arbitrage condition (37) and the short-term real interest rate (38), I find the Euler equation of the economy in the flexible-price equilibrium, given by

$$u'(C_0) = f'(Y - C_0) \cdot \hat{\beta} \cdot u'[C_1(0)] \cdot \left\{ \phi + (1 - \phi) \cdot R \cdot \frac{\beta \cdot u'[C_2(1)]}{u'[C_1(0)]} \right\}. \quad (41)$$

Define the level of implicit taxation on the banking system due to the reserve requirement as

$$\tau \equiv \rho \cdot \frac{i - i^B}{1 + i}. \quad (42)$$

Consumer's first-order condition with respect to capital goods and deposits combined determine the consumer's investment allocation. She holds a portfolio such that:

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = \min \left\{ \frac{1 - \phi}{1 - \phi - \tau}, R^{\gamma[C_2(1)]-1} \right\} \cdot R. \quad (43)$$

A higher level of taxation on banks, which is passed on to depositors as lower interest on deposits, incentivises consumers to hold fewer deposits in their portfolio, although they are imperfectly insured against liquidity shocks. In the absence of taxation, consumers decide to fully insure themselves, in the same way as the social planner would decide. Notice that the level of liquidity risk to which consumers are exposed cannot be worse than the case without deposits, where $C_2(1) = R \cdot C_1(0)$.

Market-clearing conditions combined with all the agents' budget constraints imply that the resource constraint holds as follows:

$$(1 - \phi) \cdot C_2(1) = R \cdot [f(Y - C_0) - \phi \cdot C_1(0)]. \quad (12)$$

Consumers have no incentive to consume when they do not enjoy consumption. Hence, we have that in equilibrium

$$C_1(1) = C_2(0) = 0. \quad (13)$$

In terms of optimal monetary policy, the first-best efficient allocation, as per definition 1, can be implemented by setting the interest on reserves equal to the interest rate on interbank loans. This is a version of the Friedman rule. Paying the market interest rate on bank reserves implements the efficient allocation by eliminating the distortionary taxation implied by the reserve requirement.

In the flexible-price equilibrium, the level of the nominal interest rate prevailing in the interbank market does not matter at all for the allocation. Monetary policy conducted by changing the interbank rate has no real effects, because prices change to ensure market clearing for any level of the interbank rate. The short-term real interest rate in the flexible-price equilibrium, Q^{-1} , is determined by the supply curve for capital goods and market clearing condition, according to equation (38). I define this short-term real interest rate, which is consistent with market clearing, the natural short-term real rate of interest.

Definition 3 (Natural short-term real rate of interest).

Consider flexible-price equilibrium outcomes $\{Q, C_0, C_1(\theta), C_2(\theta), \tau\}_\theta$, given by equations (41), (42), (43), (12) and (13).

Define the natural short-term real rate of interest r^n as equal to the short-term real return on capital in the flexible-price equilibrium,

$$1 + r^n \equiv Q^{-1} = 1 + r^n(\tau, \hat{\beta}). \quad (44)$$

The natural short-term interest rate is a function of the extent of financial repression operated by the government through the reserve requirement, τ , and of the consumer's time-0 discount factor. Financial repression worsens the risk-reward profile of saving and therefore leads to an increase in the natural short-term real interest rate. An increase in

the discount factor means that consumers are more patient. Hence, for markets to clear the short-term real interest rate must fall.

In the sticky-price equilibrium, the natural short-term real rate of interest becomes an important concept. If prices are sticky, they cannot change to make markets clear and guarantee full employment. Since the natural short-term real rate of interest is what the economy needs for markets to clear, it is the short-term real interest rate that a benevolent monetary authority must attempt to implement.

5.2 Sticky-Price Equilibrium

I define an equilibrium with sticky prices in order to study optimal monetary policy. Nominal rigidities are necessary for interest-rate setting by monetary authorities to have real effects.

I assume a simplified notion of nominal rigidity. In the short run, prices are fixed with $P_1 = P_0 = \bar{P}$. Since prices do not adjust to ensure market clearing, demand is not generally equal to supply. It is up to monetary policy to manage demand so that there are no spare resources.

The sticky-price equilibrium is formalised as follows:

Definition 4. *Given policy $\{i^B, i\}$ with $i^B \leq i$, the sticky-price equilibrium consists of quantities $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j), \Pi_f, K, I, \Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, M, K_g, T\}_{j,k,\theta_j}$ and prices $\{P_0, P_1, Q, d_{k,0}, d_{k,1}\}_{k \in [0,1]}$ such that*

1. *The representative capital-producing firm chooses $\{\Pi_f, K, I\}$ to maximise its profits, Π_f , defined by equation (14) subject to (1).*
2. *Consumer j chooses $\{C_{j,0}, K_j, D_{j,k}, C_{j,1}(\theta_j), C_{j,2}(\theta_j), L_j(\theta_j), W_{j,k}(\theta_j)\}$ to maximise (15) subject to budget constraints (16), (17), (18) and non-negativity constraints*

$$C_{j,t}(\theta_j) \geq 0 \quad \forall t, \theta_j. \quad (45)$$

3. *Bank k chooses $\{\Pi_k, B_k, Cu_k, F_k, K_k, D_k, L_k, d_{k,0}, d_{k,1}\}$ to maximise its profits, Π_k , subject to budget constraints (23), (25), (26), the reserve requirement (24), demand for deposits given by equations (21) and (27), and non-negativity constraints*

$$(B_k, Cu_k, K_k) \geq 0. \quad (46)$$

4. *$\{M, K_g, T\}$ are such that the government's budget constraints (28) and (29) hold.*

5. *Prices are sticky with*

$$P_0 = P_1 = \bar{P}. \quad (47)$$

6. *The goods market has*

$$\int_0^1 C_{j,0} dj + I = Y. \quad (48)$$

7. The market for reserves clears with

$$\int_0^1 B_k + Cu_k dk = M. \quad (49)$$

8. The market for interbank loans clears with

$$\int_0^1 F_k dk = 0. \quad (50)$$

9. The market for capital goods clears with

$$\int_0^1 K_j dj + \int_0^1 K_k dk + K_g = K. \quad (51)$$

Rationing takes place when at the going price there is not enough demand to absorb all the goods supplied, because the return on saving is too high.⁹ The role of monetary policy is to adjust its interest rates in order to discourage consumers from saving excessively, because this leads to rationing. However, the presence of a lower bound on the interbank rate may make it impossible for the central bank to offset large adverse demand shocks. In these cases, which I call liquidity traps, rationing takes place in equilibrium.

From here on, I drop the k index for banks, since banks behave symmetrically. Moreover, I drop the index j for consumers realising that heterogeneity between consumers is uniquely due to different realisations of a consumer's liquidity shock, θ .

There is a zero lower bound on the interbank rate, because of the option of exchanging bank reserves one-for-one with currency, given by

$$i \geq 0. \quad (52)$$

If the interbank rate was set below zero, then all banks would want to borrow and hold the borrowings as currency. This is inconsistent with market clearing in the interbank market. In the equilibrium with sticky prices, the lower bound restricts the allocations that monetary policy can attain.

The consumer's first-order condition with respect to direct holdings of capital goods and the bank's arbitrage condition (37) give us the economy's Euler equation

$$u'(C_0) = (1+i) \cdot \hat{\beta} \cdot u'[C_1(0)] \cdot \left\{ \phi + (1-\phi) \cdot R \cdot \frac{\beta \cdot u'[C_2(1)]}{u'[C_1(0)]} \right\}. \quad (53)$$

The equilibrium deposit contract is the same as in the flexible prices equilibrium, described in lemma 1. Substituting this in the consumer's first-order condition with respect to bank deposits and using the first-order condition with respect to capital goods, we find the consumer's time-0 portfolio allocation. It is such that

$$\frac{u'[C_1(0)]}{\beta \cdot u'[C_2(1)]} = \min \left\{ \frac{1-\phi}{1-\phi-\tau}, R^{\gamma[C_2(1)]-1} \right\} \cdot R. \quad (43)$$

⁹Buyers are rationed. Sellers do not consume the part of their endowment that they are unable to sell, because by assumption consumers do not like their own endowment and thus trade is necessary.

Consumers hold enough deposits to fully insure themselves against liquidity risk. τ , defined in equation (42), is the effective tax on bank deposits caused by the reserve requirement. A burdensome reserve requirement, which implies lower interest rates on deposits, makes consumers hold fewer deposits in their asset portfolio and thus exposes them to more liquidity risk.

Market-clearing conditions imply that at time 1 and at time 2 consumption satisfies the intertemporal budget constraint

$$(1 - \phi) \cdot C_2(1) = R \cdot [f(I) - \phi \cdot C_1(0)]. \quad (54)$$

And consumers do not consume in time periods when they do not enjoy consumption, so that

$$C_1(1) = C_2(0) = 0. \quad (13)$$

The level of investment in the economy, I , is determined by the interbank rate via the supply curve for capital goods. This is true, unless an excessively low interbank rate implies that demand for consumption goods at time 0 is greater than supply. In this case, investment is limited by the availability of consumption goods to invest.

$$f'(I) = \max\{1 + i, f'(Y - C_0)\}. \quad (55)$$

Lemma 2. *In the sticky-price equilibrium, the allocation $\{I, C_0, C_1(\theta), C_2(\theta), \tau\}_\theta$ is given by equations (53), (42), (43), (54), (13) and (55). Policy instruments $\{i^B, i\}$ are subject to the following restrictions:*

$$i \geq 0, \quad (56)$$

$$i \geq i^B. \quad (57)$$

6 Optimal Monetary Policy

Policymakers choose policy instruments $\{i^B, i\}$ in order to maximise aggregate welfare in the sticky-price equilibrium.

It is useful to define liquidity traps in this setting, because optimal monetary policy differs depending on whether the economy has fallen in the liquidity trap or not.

Definition 5. *If $r^n(0, \hat{\beta}) < 0$, then the economy is in the liquidity trap.*

It can be shown that there are realisation of $\hat{\beta}$ such that the economy is in the liquidity trap. A high realisation of $\hat{\beta}$ implies that consumers are patient in their consumption decisions. Thus, a low real interest rate is required for demand to fully absorb the supply of goods at time 0. If a negative interest rate is required, then we say that the economy is in the liquidity trap.

Lemma 3. *Consider the function $r^n(\tau, \hat{\beta})$ in definition 3. There exists a threshold $\bar{\beta}$ such that, if $\hat{\beta} > \bar{\beta}$, then $r^n(0, \hat{\beta}) < 0$.*

The presence of the lower bound on the interbank rate makes the liquidity trap a special economic contingency for policymakers.

In the first subsection, I discuss optimal monetary policy when the economy is not in the liquidity trap. Then, I move on to the case of an economy with very high propensity to save and study the implications for optimal monetary policy.

6.1 Out of the Liquidity Trap

When a positive level of the interest rate makes the market for consumption goods clear, then the monetary authority should set the interbank rate at that level. Simultaneously, by setting the interest on bank reserves at the same level, the central bank can implement full liquidity-risk insurance in the economy.

Proposition 1. *If the economy is not in the liquidity trap with $r^n(0, \hat{\beta}) \geq 0$, then the monetary authority can implement the first-best efficient allocation with*

$$i^B = i = r^n(0, \hat{\beta}). \quad (58)$$

Proof. □

There is no tension between making sure none of the endowment is wasted and not interfering with the provision of liquidity-risk insurance by banks. The central bank should follow the Friedman rule and not tax holders of liquid assets. Demand management can be accomplished exclusively by setting the interest rate in the interbank market. In summary, outside of the liquidity trap it is fine to study optimal interest-rate setting in the absence of maturity transformation.

6.2 Liquidity Trap

In this paragraph, I study the optimal setting of the interest on reserves in the liquidity trap. As a consequence of large adverse demand shock hitting the economy, a negative short-term real interest rate is necessary to ensure enough demand to fully absorb supply. In this case, the lower bound on the interbank rate represents a constraint on the central bank's ability to ensure full utilisation of the economy's resources. I show that in the liquidity trap the central bank faces a trade-off between liquidity-risk insurance and ensuring full employment. This is because banks that perform maturity transformation encourage consumers to save. And in the liquidity trap insufficient demand is caused by an excessive propensity to save from consumers.

First of all, the lower bound on the rate of interest prevailing in the interbank market is binding in the economy's liquidity trap. That is, it is optimal for the central bank to set the interbank rate to zero, whenever the economy is in the liquidity trap.

Lemma 4. *For any level of $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$, if the economy is in the liquidity trap as per definition 5, then it is optimal to set $i = 0$.*

Proof. Please refer to appendix B. □

It is hardly surprising that in the liquidity trap it is optimal for the central bank to hold the interbank rate at the lower bound.

Consider $\tau = 0$ so that maturity transformation leads to full liquidity-risk insurance, as according to equation (43). In this case, since the interbank rate is larger than the natural short-term real interest rate, there is rationing in the goods market. An alternative policy is to set $\tau > 0$. This reduces liquidity-risk insurance to a suboptimal level. On the

other hand, less liquidity-risk insurance reduces consumers' incentive to save, as according to the Euler equation (53), which can reduce wasted resources. We learn two things about an economy in the liquidity trap from this reasoning: 1) the central bank cannot attain the first-best efficient allocation and 2) there is an interaction between aggregate demand and maturity transformation that the central bank may want to exploit.

The interaction between demand management and maturity transformation is represented by the *IMP* schedule in the cartesian plane of figure 2. Point *A* is the combination of liquidity-risk insurance and time-0 consumption that prevails if the central bank sets $i^R = 0$ and thus implements perfect liquidity-risk insurance. The economy moves along the *IMP* constraint to the left as the interest on reserves is cut. Lower interest on reserves reduces the attractiveness of deposits, as banks are effectively taxed. Hence, consumers substitute their wealth away from deposits into direct capital holdings. However, deposits and capital are not perfect substitutes, in that the former provides liquidity-risk insurance. It follows that the consumer partially substitutes away from saving overall and increases her current consumption. The quasi concavity of the *IMP* curve is given by an income effect, which pulls in the other direction. Consumers are made poorer by the reduction in the attractiveness of investment opportunities. This increases the value of saving. At the point where the implementability curve peaks, the income effect and the substitution effect perfectly cancel each other out. To the left of the peak, the income effect dominates. However, the range of liquidity-risk insurance where the income effect dominates is not important for the optimal monetary policy exercise, because it is always suboptimal for monetary policy to move the economy to it.

The central bank's role is to maximise aggregate welfare with the policy instruments at its disposal. Aggregate welfare is represented on the Cartesian plane by a family of indifference curves, labeled *IND*. Aggregate welfare is increasing with current consumption and with liquidity-risk insurance. If $u'[C_1(0)] = R \cdot \beta \cdot u'[C_2(1)]$, consumers are satiated in liquidity-risk insurance. This means that, if they have perfect liquidity-risk insurance, consumers are willing to reduce it marginally in exchange for any strictly positive increase in current consumption, however small. This is captured in the figure by the flatness of the indifference curves at that point.

Optimality requires that the rate at which consumers would trade off liquidity-risk insurance for more consumption remaining equally well off be equal to the rate at which the central bank can increase consumption by decreasing liquidity-risk insurance. This is given by the tangency point of curve *IND* and *IMP*, point B in figure 2.

Proposition 2 (Optimal Monetary Policy in Liquidity Trap).

If the economy is in the liquidity trap, optimal policy prescribes $i = 0$ and $i^B < 0$. It implements a second-best allocation.

Proof. Please refer to appendix B. □

Point B in figure 2, which represents the second-best equilibrium allocation, always features partial liquidity-risk insurance. So, setting the interest on reserves strictly below the lower bound on the interbank rate is optimal. Notice that this monetary policy does not transmit to the economy through the conventional intertemporal substitution channel via

a reduction in the real interest rate. It solely transmits by making deposits less attractive for consumers, relative to direct holdings of capital.

7 Conclusion

In this paper, I develop a monetary model with two important characteristics: multiple interest rates, which monetary policy controls, and a meaningful banking sector. With reference to the former, I explicitly model the interbank rate, which is the conventional instrument of monetary policy, and the interest on reserves separately. And as regards the latter, banks are modelled as maturity transformers, in accordance with the framework of [Diamond and Dybvig \(1983\)](#).

I have four main findings in the paper. First, I show that the lower bound does not apply to the interest on reserves. A first-pass explanation for this is that the central bank can decide by decree to pay whatever interest on bank reserves, while the interbank rate is the equilibrium price that clears the interbank market. The lower bound emerges as equilibrium requirement on the latter because of the presence of currency. Nonetheless, if reserves and other assets are perfect substitutes as in the canonical New Keynesian model, changing the interest on reserves per se has no macroeconomic effect.

Second, I show that reserves and other assets become imperfect substitutes if banks perform maturity transformation à la [Diamond and Dybvig \(1983\)](#). From the consumer's perspective, deposits and other assets are not perfect substitutes because of the benefits of maturity transformation. Since reserves are necessary to supply redeemable deposits, bank reserves also become imperfect substitutes of other assets. Thus, a reduction in the interest on reserves, which leaves the interbank rate unchanged, increases aggregate demand.

Third, I find that setting a negative interest on reserves in order to boost aggregate demand in the liquidity trap involves an interesting trade-off with the preservation of a fully functional banking system. A lower interest on reserves transmits to the deposit rate. Thus, consumers respond by moving their wealth out of deposits into direct asset holdings. Such disintermediation is detrimental to welfare in this setting, because deposits provide valuable liquidity-risk insurance. In summary, stimulus can be provided to the economy by means of negative interest on reserves only against the backdrop of a weakening banking system.

The last and most important finding of the paper is that, when the interbank rate is constrained by the zero lower bound, optimal monetary policy prescribes a strictly negative interest on reserves. In other words, I find that, when demand is insufficient, there is always some space to stimulate the economy by cutting the interest on reserves below the interest rate prevailing in the money market without damaging the banking sector excessively.

In reality, central banks implement negative interest rate policies by cutting the interest on reserves without knowing where the lower bound on the interbank rate is. They do not know when the cuts to the interest on reserves will stop transmitting to the money market. A discussion has developed over the extent to which central banks should therefore be prudent in lowering their interest on reserves ([Dell'Ariccia et al., 2017](#)). The result of my paper can be read as policy recommendation not to be prudent,

because lowering the interest on reserves below the lower bound on the interbank rate is the optimal monetary policy.

Moreover, my theory contributes a relevant and measurable indicator for the health of the banking sector, which could guide monetary authorities in the implementation of negative interest rates. The degree of liquidity-risk insurance, which represents the effectiveness of the banking sector at providing liquidity, is expressed in terms of observables and therefore can in principle be measured. The theory predicts that negative interest rates lead to a decline in liquidity-risk insurance, and makes clear that, while a moderate reduction is necessary in order to stimulate the economy, an excessive reduction is undesirable. Therefore, monitoring this measure would allow to adjust interest-rate setting in accordance with the level of stress that it imposes on the banking system.

This paper is the first to study the role of the interest rate on bank reserves in the liquidity trap of a model with maturity-transforming banks. Maturity transformation is an important aspect of banking. However, it is by no means the only one. Future research should enrich the modelling of the banking sector, on the asset side ([Brunnermeier and Koby, 2017](#); [von Thadden, 1997](#)) and on the liability side with capital constraints. This will let us establish quantitatively the extent to which bank characteristics matter for the effectiveness of negative interest rates.

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A Figures

Figure 1: Interest rate on excess reserves, 2012-2017

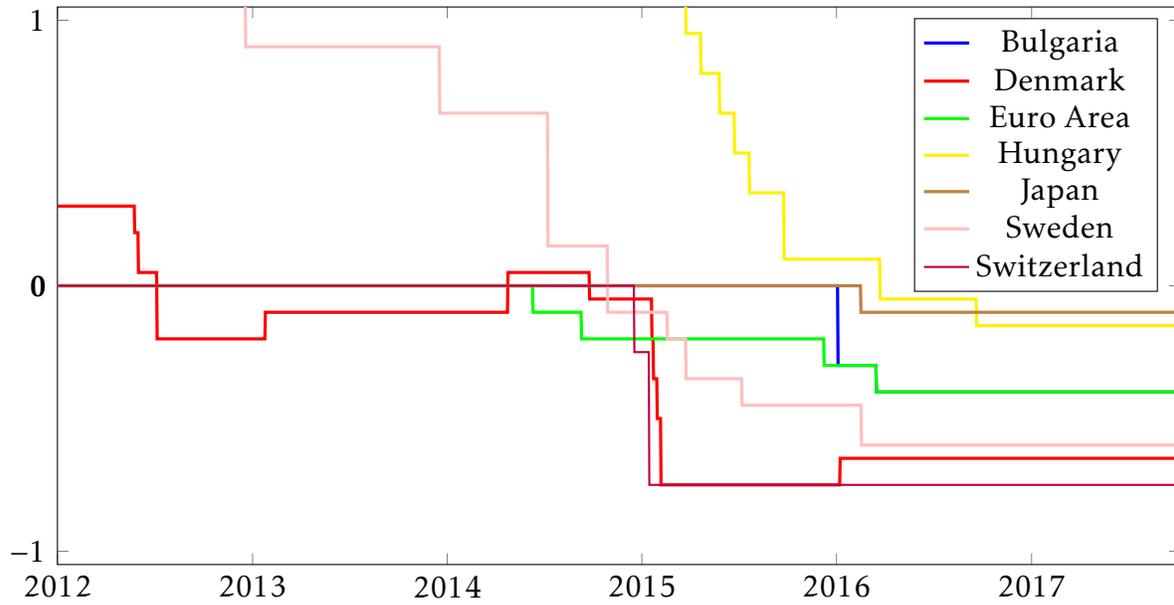
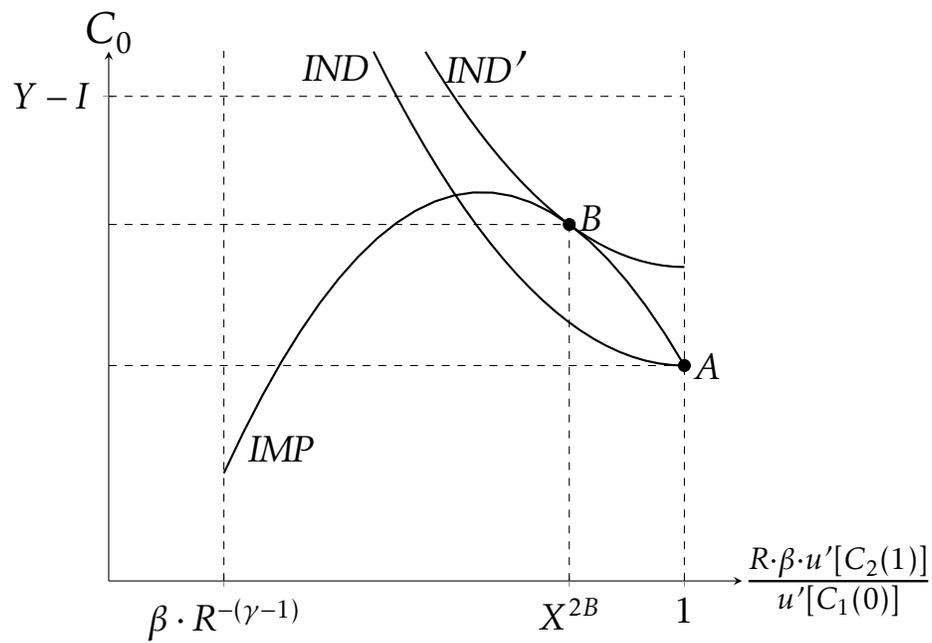


Figure 2: The second-best problem



B Proofs

Lemma 1. Define $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$. In equilibrium, the prevailing deposit contract $\{d_0, d_1\}$ is given by:

$$1 + d_0 = (1 + i) \cdot \frac{P_0}{P_1} \cdot \frac{1 - \tau}{\phi} \quad (39)$$

$$1 + d_1 = 0 \quad (40)$$

Proof. Consider the flexible-price equilibrium allocation $\{C_0, C_1(\theta), C_2(\theta), \tau\}_\theta$ governed by equations (41), (42), (43), (12) and (13).

By Bertrand competition, banks make zero profits. It follows from banks' budget constraints, (23), (25), (26), the reserve requirement, (24, and non-negativity of capital holdings, $L \leq K$, that feasible deposit contracts are restricted to

$$1 + d_0 = (1 + i) \cdot \frac{P_0}{P_1} \cdot (1 - \tau) \cdot \frac{R}{\phi \cdot R + (1 - \phi) \cdot (1 + d_1)}, \quad (59)$$

with

$$1 + d_1 \geq 0. \quad (60)$$

I show that there is no deposit contract that is feasible and that consumers prefer in equilibrium.

Take the deposit valuation equation (20) and substitute in the equilibrium allocation and feasibility conditions for deposit contract. We have that

$$V = (1 + i) \cdot \frac{P_0}{P_1} \cdot (1 - \tau) \cdot \frac{R}{\phi \cdot R + (1 - \phi) \cdot (1 + d_1)} \cdot \hat{\beta} \cdot [\phi \cdot Z + (1 - \phi) \cdot (1 + d_1) \cdot \beta] \cdot u'[C_2(1)], \quad (61)$$

with

$$Z = \min \left\{ \frac{1 - \phi}{1 - \phi - \tau}, R^{\gamma[C_2(1)]-1} \right\} \cdot \beta \cdot R. \quad (62)$$

The value of the deposit contract is continuously non-increasing in d_1 . Hence, there is no feasible deposit contract that consumers strictly prefer to the deposit contract with $1 + d_1 = 0$. □

Lemma 4. For any level of $\tau \equiv \rho \cdot \frac{i-i^B}{1+i}$, if the economy is in the liquidity trap as per definition 5, then it is optimal to set $i = 0$.

Proof. Consider the sticky-price equilibrium illustrated in lemma 2 with τ given.

From equation (55), a reduction in i increases investment I . Through equation (54), the increase in investment increases $C_1(0)$ and $C_2(1)$.

From equation (53), a reduction in i leads to an increase in C_0 .

Holding τ constant, an increase in i weakly increases consumption in all time periods. Hence, it is unambiguously (weakly) welfare improving.

It follows that in the liquidity trap, for any level of τ , it is optimal to hold the interbank rate against the lower bound. □

Proposition 2 (Optimal Monetary Policy in Liquidity Trap).

If the economy is in the liquidity trap, optimal policy prescribes $i = 0$ and $i^B < 0$. It implements a second-best allocation.

Proof. Consider the graphical representation of the second-best problem given in figure 2.

The implementability constraint, labelled by *IMP*, is given by equations (53), (54) and (55) with $i = 0$ because by lemma 4 this is optimal. The first-order derivative of time-0 consumption C_0 with respect to the level of liquidity-risk insurance $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}$ is given by

$$\frac{dC_0}{d\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}} = \frac{\phi \cdot (1 - \phi) \cdot \hat{\beta} \cdot u'[C_1(0)]}{u''[C_0]} \cdot \left(\frac{R \cdot \frac{\gamma[C_2(1)]}{\gamma[C_1(0)]} \cdot \frac{C_1(0)}{C_2(1)} \cdot \frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} - 1}{\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} \cdot \left\{ \phi \cdot R \cdot \frac{\gamma[C_2(1)]}{\gamma[C_1(0)]} \cdot \frac{C_1(0)}{C_2(1)} + 1 - \phi \right\}} \right). \quad (63)$$

Notice that at $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} = 1$, with full liquidity-risk insurance, the derivative is negative and that as liquidity-risk insurance decreases the first-order derivative of the *IMP* schedule increases. Thus, the graphical representation is accurate.

The indifference curve, labelled *IND*, gives bundles of liquidity-risk insurance and time-0 consumption such that expected utility is constant. The first-order derivative of time-0 consumption C_0 with respect to level of liquidity-risk insurance $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}$ on the schedule is given by

$$\frac{dC_0}{d\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}} = \frac{\phi \cdot (1 - \phi) \cdot \hat{\beta} \cdot \{u'[C_1(0)]\}^2}{u''[C_1(0)] \cdot u'(C_0)} \cdot \frac{1}{\phi \cdot R \cdot \frac{\gamma[C_2(1)]}{\gamma[C_1(0)]} \cdot \frac{C_1(0)}{C_2(1)} + 1 - \phi} \cdot \frac{1 - \frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}}{\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]}}. \quad (64)$$

Notice that the derivative of the indifference curves is equal to zero if $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} = 1$. This means that at perfect liquidity-risk insurance the consumer is satiated in liquidity.

It follows that for the central bank setting $\frac{\beta \cdot R \cdot u'[C_2(1)]}{u'[C_1(0)]} = 1$ is inefficient. While a marginal increase in liquidity risk does not reduce expected utility, the resulting increase in current consumption does. \square