Optimal Negative Interest on Reserves

The Maturity-Transformation Channel of Monetary Policy Transmission[a]

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Abstract

This paper studies the role of the interest rate on bank reserves in monetary policy. For this purpose, I consider a model with a banking sector, where the transmission of monetary policy to the interbank market is explicitly modelled via open market operations and the interest on reserves. First, I show that the lower bound applies to the interbank rate (i.e., the conventional monetary policy instrument) but not to the interest on reserves. Nonetheless, if deposits and other assets are perfect substitutes, changes in the interest on reserves per se have no macroeconomic effect. I model banks according to the classical maturity-transformation framework of Diamond and Dybvig (1983). Banks transform the available assets into deposits, which better suit the consumers' liquidity needs, and by doing so they promote consumers' saving behaviour. I show that maturity transformation makes deposits and other assets imperfect substitutes. This creates the maturity-transformation channel of monetary policy, whereby a reduction in the interest on reserves, which leaves the interbank rate unchanged, increases aggregate demand. However, there is a downside to boosting demand through this channel: consumers also respond by moving part of their wealth away from deposits into direct asset holdings. Such disintermediation is detrimental to welfare in this setting, because deposits provide valuable liquidity-risk insurance. An interesting policy trade-off emerges between preserving a fully functional banking system and stimulating demand. The paper's main finding is that, when the interbank rate is constrained by its lower bound, optimal monetary policy prescribes an interest on reserves strictly below the lower bound on the interbank rate.

Keywords: Interest on reserves, lower bound, maturity transformation, unconventional monetary policies.

JEL Codes: E43, E52, E58, G21.

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1 Introduction

In the aftermath of the Great Recession, the interest rate on bank reserves has become the active monetary policy tool in advanced economies. Central banks increased the supply of reserves massively. This turned the monetary system into a floor system, in which the stance of monetary policy is determined by the interest rate on reserves rather than by the scarcity of bank reserves (i.e., open market operations).[1] Also, the implementation of negative interest rate policies, performed by several advanced-economy central banks, required the active use of the interest on reserves.[2]

In light of this development in the practice of monetary policy, in this paper I study the role of the interest on reserves in pursuing the central bank's policy objectives. A literature has recently developed on the role of reserves and of the interest on reserves in monetary policy (Kashyap and Stein, 2012; Ennis, 2014; Reis, 2016). However, such literature abstracts from negative interest rates and the challenges to monetary policy posed by the lower bound on nominal interest rates. A separate literature studies the effectiveness of negative interest rates in pursuing macroeconomic objectives when an economy is in the liquidity trap (Rognlie, 2016; Brunnermeier and Koby, 2017). However, as most of the literature on negative interest rates, their analysis concentrates on only one interest rate, the interbank rate. This simplification is more restrictive than usual when negative interest rates are the subject, because it does not allow the analysis to focus on the key issue of the transmission of monetary policy to the money market. I provide a theoretical contribution to the literature on negative interest rates, which I expand by including insights from the literature on reserves.

In the model, I explicitly model monetary policy as a choice of interest on reserves and open market operations, which determines the interbank rate. This a more realistic description of the economy and of monetary policy. Central banks influence multiple interest rates. This explicit modelling of monetary policy transmission allows me to study whether the lower bound on nominal interest rates constrains different interest rates in different ways, and if this has implications for monetary policy in the liquidity trap. The contribution of my paper moves the current debate on negative interest rates closer to the original literature on the liquidity trap, as developed from the seminal paper by Krugman (1998), which is founded on the idea that the interbank rate cannot fall below a lower bound.

The lower bound on the interbank rate represents the idea that a low enough interest on reserves will fail to ease monetary policy via the conventional channel, by lowering the interbank rate and thus, for example, reducing the rewards from saving. In policy circles the question is not whether there is a lower bound, but how to estimate it (Alsterlind et al., 2015; Bech and Malkhozov, 2016; Dell'Ariccia et al., 2017). A recently disclosed Federal Reserve internal memo (Burke et al., 2010) estimates that the federal funds rate can at

[1]For example, the Federal Reserve in the last quarter of 2008 alone issued \$800bn in bank reserves, amounting to a nine-fold increase in the supply of reserves. It did not have the authority to pay interest on bank reserves. So, the Emergency Economic Stabilization Act of 2008 granted it such authority on 1 October 2008. This allowed the central bank to continue controlling the fed funds rate, as banks became satiated in reserves.

[2]Please refer to figure 5 in appendix E for data on the implementation of negative interest on reserves.

most be lowered to -35 basis points.[3] Considering that the current interbank rate in the Euro Area is -0.35%, hitting the lower bound is a real possibility in case further monetary easing were desirable.[4] Therefore, a full evaluation of the desirability of negative interest rate policies must take this eventuality into account.[5]

The interest on reserves is not constrained by a lower bound in the same way as the interbank rate is. A first-pass explanation for this is that the central bank can decide by decree to pay whatever interest on bank reserves, while the interbank rate is the equilibrium price that clears the interbank market. The central bank controls the interbank rate only indirectly by carrying out open market operations and by setting the interest on reserves, and I show that the lower bound emerges as an equilibrium requirement due to the existence of zero-interest-paying physical currency.[6]

Nonetheless, this does not imply that the central bank can ease monetary policy just by lowering the interest on reserves, once the interbank rate is stuck at the the lower bound. Consider the canonical New Keynesian model, where the banking sector is just a veil behind which consumers lend to firms. In such model, lowering the interest on reserves per se would not be expansionary, because, even if banks were forced to hold bank reserves, consumers would move all of their savings out of deposits into the money market if the cut in the interest on reserves led to a reduction in the deposit rate. In other words, in such framework deposits and bonds are perfect substitutes from the consumers' perspective and therefore the interest on reserves has a macroeconomic role only insofar as it steers the interest rate in the bond market. So, in order to meaningfully study the macroeconomic effects of the interest on reserves, we need a theory of banking which generates a demand for deposits.

The role of banks in the model that I develop is based on the classical framework by Diamond and Dybvig (1983) on maturity transformation. Banks are depository institutions. Consumers are subject to idiosyncratic liquidity shocks, which may force them to wind up their investments early and thus forego the liquidity premium on their assets. The combination of liquidity shocks and liquidity premium gives rise to liquidity risk for consumers. Banks emerge in the model to provide insurance against liquidity risk. They do so by issuing redeemable deposits and holding long-term assets. In other words, banks perform maturity transformation, which means that even short-term depositors enjoy to some extent the higher yields on the long-term assets that are held by their bank. The key feature of this model is that banks facilitate saving behaviour, because they provide consumers with assets that are more liquid than the assets available in direct asset markets. This implies that deposits and direct holdings of assets are not perfect substitutes from the perspective of consumers.

An influential strand of the literature on negative interest rates also has its focus on

[3] Burke et al. (2010) is the only publicly accessible paper that reports the methodology with which the lower bound on the interbank rate is estimated. Viñals et al. (2016) reports estimates by IMF staff ranging from -75 basis points to -200 basis points, but without disclosing the analysis.

[4] The Eonia rate is the overnight interbank rate in the Euro Area. It is -0.35% as of 6 November 2017. Concurrently, the interest on excess reserves in the Euro Area is -0.4%.

[5] Bernanke (2016) makes this point to argue that negative interest rate policies are unlikely to have large stimulative effects.

[6] Hicks (1937) was the first to describe the lower bound on the money-market interest rate in these terms.

the banking system, but uses a different theory of banking. It studies the interaction of a negative interbank rate with the banking system, by emphasising the asset side of banks: the key role of the banking system in their model is to invest, because banks are defined by their high productivity at making investments. They show that, if deposit rates are downwardly sticky at zero, there is a a level of the interbank rate beyond which cuts are contractionary, because they harm bank profitability and thus investment (Brunnermeier and Koby, 2017). The framework that I propose is not in contradiction with this strand of the literature. In fact, it complements it by focusing on another one of the many different roles that banks play in the economy: maturity transformation.

Since in my model deposits and direct holdings of assets are not perfect substitutes, I find that a reduction in the interest on reserves does not only lead to disintermediation. Interestingly, it also leads to a reduction in savings and thus to an increase in aggregate demand. I call this the maturity-transformation channel of monetary policy, because in the absence of a maturity-transforming banking system changes in the interest on reserves per se have no macroeconomic significance. The mechanism whereby the interest on reserves affects consumers' propensity to save runs through the deposit rate. Banks hold reserves, either because they are required to do so or because reserves are useful in the payment system among banks. Thus, a reduction in the interest on reserves translates into a lower deposit rate. The reduction in the deposit rate makes the assets available to consumers less attractive overall, because deposits and other assets are imperfectly substitutable. Hence, consumers respond both by moving their wealth out of deposits into direct holdings of assets and by saving less in total.

A benevolent central bank finds that boosting aggregate demand by exploiting the maturity-transformation channel of monetary policy is beneficial when aggregate demand is insufficient. There is however a downside to stimulating the economy by means of the maturity-transformation channel: it reduces the level of bank activity. In this economy banks provide liquidity-risk insurance, which is desirable. Therefore, an interesting policy tradeoff emerges between stimulating the economy and preserving the banking system.

The main finding of the paper is that, when the lower bound on the interbank rate is binding and the economy needs further easing, it is optimal to lower the interest on reserves strictly below the interbank rate. This boosts employment but also reduces the effectiveness of the banking system. However, the point at which the negative effect of the latter outweighs the benefits of the former is strictly below the lower bound. When the economy is in the liquidity trap with insufficient demand, the central bank should exploit the maturity-transformation channel of monetary policy transmission to stimulate the economy. This result is important for monetary policy in the liquidity trap, because it identifies an unconventional policy instrument that can improve the performance of the economy when further reductions in the interbank rate cannot be implemented.

Rogoff (2017) identifies the lower bound on the conventional monetary policy instrument (i.e., the interbank rate) as the key constraint currently facing central bankers. Moreover, there is a growing literature (Karabarbounis and Neiman, 2013; Summers, 2014; Carvalho et al., 2016) arguing that structural factors are pushing down the real interest rate, resulting in a higher probability of hitting the lower bound in the future. This suggests that the literature on unconventional monetary policy, in which this paper fits, is relevant and will remain relevant in the future.

1.1 Related literature

Unconventional monetary policies, defined as policies within the remit of the central bank other than targeting the spot short-term interbank interest rate, have attracted a large academic literature.

This paper is closely related to a small number of papers that study negative interest rate policies. Rognlie (2016) postulates a demand for currency with a bliss point within a New Keynesian set-up and finds that transmitting a negative interest rate to the bonds market is feasible. Moreover, he finds that it is desirable when the economy experiences a liquidity trap. Brunnermeier and Koby (2017) and Eggertsson et al. (2017) study the interaction between negative rates and banks. They assume a lower bound on deposit rates and find that negative interest rate policies reduces banks' interest margin and profits. Since banks as subject to a capital constraint and are the only lenders in the economy, negative interest rate policies, which lower bank profits, are contractionary. The evidence for a lower bound on deposit rates is mixed. Bech and Malkhozov (2016) and Heider et al. (2017) find evidence for it. The latter show that retail banks in the Eurozone more reliant on deposits suffered more from the European Central Bank's negative interest rate policy. On the other side of the argument, Basten and Mariathasan (2017) show that Swiss retail banks funded with more deposits, which should have been hit harder by negative interest rate policies in the presence of a binding zero lower bound on deposit rates, increased fees on depositors more and hence did not suffer in terms of profits. Effectively, this means that negative interest rate policies were passed on to depositors. In my paper, I model the banking sector in a novel way in the context of this literature, as a maturity transformer, and I focus on the lower bound on the interbank rate, which has a long tradition in the New Keynesian liquidity trap and can be derived from the presence of paper money.

A theory of banking based on the notion of maturity transformation was first formalised by Diamond and Dybvig (1983). A subsequent theoretical literature worked on expanding the framework with general-equilibrium elements. Jacklin (1987) and Allen and Gale (2004) add financial markets and discuss the implications for the incentive compatibility of the demand deposit contract.[7] Hellwig (1994) and Farhi et al. (2009) introduce competitive forces in the banking sector.

The existence of a lower bound on the nominal interest rate prevailing in the bonds market, which acts as a constraint on monetary policy, was first hypothesised by Hicks (1937). The seminal paper for the modern literature is Krugman (1998), and a very large literature followed. An integration of the lower-bound friction with the canonical New Keynesian framework, as laid out by Woodford (2003) and Galí (2008), is reached in Eggertsson and Woodford (2003).[8] Demand shocks are commonly modelled as simple time-preference shocks within the literature. More structure on the large adverse demand shock that leads into the liquidity trap is provided by Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012), who model the adverse demand shock as a consequence of a credit crisis.

This paper finds that the interest rate on reserves has macroeconomic effects per se,

[7] See also Diamond (1997) and von Thadden (1998) on this subject. A discussion of the incentive compatibility of maturity transformation with multiple assets is available in von Thadden (1997).

[8] In Werning (2011) a similar model is presented in continuous time.

beyond the mere transmission of the monetary policy stance to the interbank rate. Bank reserves and the interest rate on reserves recently received a great deal of attention. Reis (2016) discusses monetary policy by central banks with large balance sheets. He finds that maintaining banks satiated in their reserve holdings is desirable, because it makes the choice of interest rate independent of the balance-sheet size. This frees quantitative easing for use as an independent policy instrument that can achieve independent policy goals. Cúrdia and Woodford (2011) also advocate a floor system of monetary policy on the ground that a system which sets the interest rate on reserves below the interbank rate is effectively taxing the banking sector. Eliminating such tax implements the Friedman rule and is therefore optimal. Moreover, they also find that the floor system has the benefit of making the central bank's balance-sheet size independent of the target interest rate. Thus, it can be used to react to financial-intermediation disturbances. Kashyap and Stein (2012) contrast these views by pointing out that, if short-term debt issued by the banking sector poses a systemic risk, it is optimal to set a non-zero wedge between the interbank rate and the interest rate on reserves in order to pursue macroprudential objectives. The interest-rate wedge serves as a tax, disincentivising excessive issuance of short-term debt by the banking sector. This paper's finding that optimal monetary policy in the liquidity trap requires a positive wedge between the interbank rate and the interest on reserves is similar: the interest rate wedge acts as a tax on bank deposits, discouraging excessive saving by consumers.

The macroeconomic effects of the size and composition of the central bank's balance sheet received most attention in the literature on unconventional monetary policy. This strand of the literature was started by an irrelevance result proven by Eggertsson and Woodford (2003) in the context of the canonical New Keynesian model. Subsequently, arguments were set forth for the macroeconomic importance of both the size (Clouse et al., 2003; Vayanos and Vila, 2009; Krishnamurthy and Vissing-Jorgensen, 2012) and the composition (Gertler and Karadi, 2011; Cúrdia and Woodford, 2011) of the central bank's balance sheet. My model maintains the irrelevance of the central bank's balance sheet, in order to isolate the effects of the interest on reserves.

2 The Model

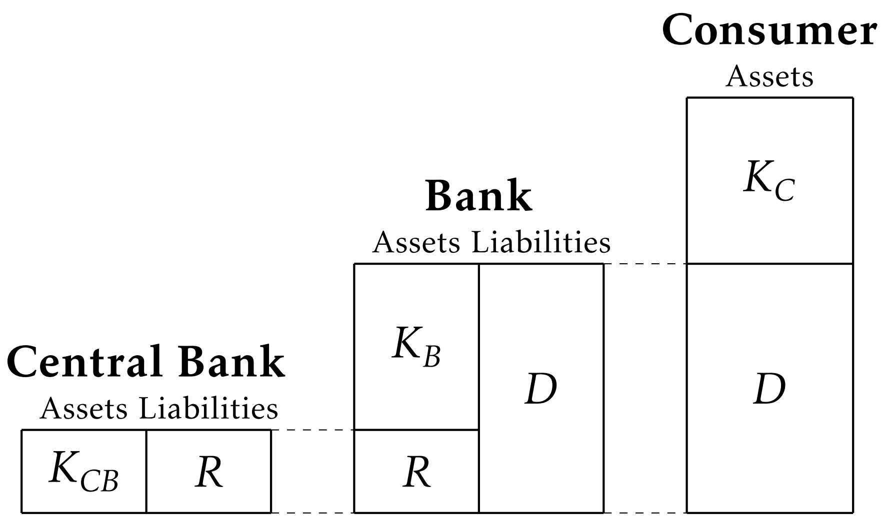
This section develops a 3-period general equilibrium model. Key elements of the model are the presence of liquidity risk for consumers, which creates a role for banks to supply deposits, and the presence of paper money, which generates a lower bound on nominal interest rates. Moreover, nominal rigidities, which are crucial to give a role to monetary policy, are introduced in section 3.

First, I describe the optimisation problem of the firms operating in the capital-production sector. Then, I describe the behaviour of consumers and banks. Last, I present monetary policy as carried out by the central bank.

2.1 Capital-Production Sector

The fundamental asset of the economy is physical capital. It is not the only asset in the economy, but production of capital goods is the only way for the economy as a whole to move resources over time. In fact, as illustrated by figure 1, physical capital backs up all other assets. Consumers save both directly in capital and in deposits supplied by banks. Deposits are backed up by the bank's assets, which are holdings of physical capital and reserves. And reserves are supplied by the central bank by means of open market operations wherein it purchases capital goods.

Figure 1: Balance Sheets.



Capital is supplied in a competitive market by firms. First, I describe the individual firm's maximisation problem and then examine the capital supply function that it implies. The properties of the capital supply function are important to ensure that the model features liquidity risk and endogenous interest rates. Please refer to appendix B for a discussion of capital supply in Diamond and Dybvig (1983) and its relation to this model.

2.1.1 Firm's maximisation problem

The market for capital goods is perfectly competitive. In every period, a unit measure of identical static firms operate within it.

Firms invest consumption goods I to produce new physical capital K according to technology

$$K = f(I), \tag{1}$$

with f twice-continuously differentiable, $f' > 0$, $f'' < 0$ and $f'(0) = \lambda < (1+r)^{-\frac{\gamma-1}{\gamma}}$. Notice that the assuptions on f imply that the technology features decreasing returns to scale and that the rate of transformation of consumption goods into capital goods is smaller than 1 for any weakly positive level of production.[9]

In each period, firms statically maximise their profits Π_F, given by

$$\Pi_F = Q \cdot K - I, \tag{2}$$

[9]The assumption $\lambda < (1+r)^{-\frac{\gamma-1}{\gamma}}$ means that the liquidity premium on capital is sufficiently large. This ensures that banks and financial markets coexist in the equilibrium.

where Q is the price of physical capital in terms of consumption.

The formal definition of the firm's maximisation problem is given by

Definition 1 (**Firm's maximisation problem**).
Taking as given the price Q, each firm chooses the quantity of consumption goods to invest I, the quantity of capital to produce K, and profits Π_F. Choices are made to maximise the firm's profits Π_F subject to the technology (1) and the definition of profits (2).

2.1.2 Capital Supply

In perfect competition, firms produce up to the point at which the marginal cost of production is equal to the price of capital Q, so that

$$Q = \frac{1}{f'(I)}. \tag{3}$$

Equation (3), in combination with the capital-production technology (1), determines a supply curve for physical capital

$$K = K^S(Q). \tag{4}$$

Lemma 1 (**Properties of capital supply**).
The capital supply function has the following properties:

1. $\frac{\partial K^S(Q)}{\partial Q} \in (0, +\infty)$,
2. $K^S(Q) \geq 0 \iff Q > (1+r)^{\frac{\gamma-1}{\gamma}}$.

Proof. Please refer to appendix D. □

A graphical representation of the capital supply curve is available in appendix E in figure 6. It is strictly increasing in the price of capital at a finite rate. This is a direct implication of strictly decreasing returns in the production of capital goods. The latter assumption can be interpreted as firms having to pick investment opportunities which require for example more costly monitoring as they scale up. This property of capital supply is necessary for the framework to meaningfully accommodate monetary policy: only a finite and strictly positive price-elasticity of capital supply delivers a finite reduction in investment and a finite increase in the yield on capital investment as a consequence of monetary tightening. Perfect elasticity would give infinite responses and perfect inelasticity would give no response at all, as discussed in detail in appendix B.

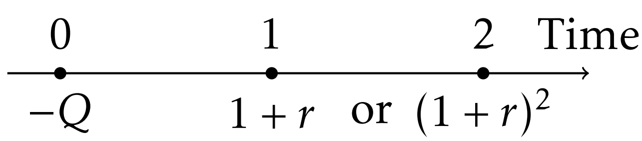
According to the second property of lemma 1, weakly positive levels of capital production are associated with prices of capital strictly greater than 1. Since demand for capital is non-negative, this means that in equilibrium the price of capital is always strictly greater than 1. This property of the supply of capital has two roles. First, it makes sure that agents who do not need consumption goods in the current period find it optimal not to liquidate their capital goods but to hold on to them, because capital can be transformed into consumption goods one-for-one but the price of purchasing capital goods with consumption goods Q is strictly greater than 1. Second, it is a crucial ingredient for liquidity risk to be a feature of the model. Liquidity risk is the risk of forgoing the liquidity premium on one's assets if one has to liquidate them too early. In the next subparagraph, I describe the payoffs from holding capital and show that $Q > 1$ is necessary and sufficient in this model to generate a liquidity premium on capital.

2.1.3 Capital Payoffs and the Liquidity Premium

For the model to feature liquidity risk it is essential that the payoff structure of capital is characterised by a strictly positive liquidity premium. This means that the yield from holding capital must be increasing with the time for which the asset is held.[10] In other words, long-term holdings of assets are more profitable than short-term holdings.

A unit of physical capital costs Q in consumption terms. Then, it grows at a constant rate r in each period. Upon liquidation at time 1 or at time 2, the holder earns income from her investment by transforming the capital goods in consumption goods one for one. This payoff structure of capital in consumption terms is illustrated in figure 2.

Figure 2: Capital payoffs



The yield on capital depends on the timing of liquidation. Using the definition of yield given by equation (5), one can see that the yield on capital held for two periods is $y = \frac{1+r}{\sqrt{Q}}$ and the yield on a capital investment liquidated after one period is $y = \frac{1+r}{Q}$. I define the liquidity premium LP as the difference between the yield on capital held for 2 periods and the yield on capital held for one period,

$$LP = \frac{1+r}{Q} \cdot \left(\sqrt{Q} - 1\right). \tag{6}$$

The liquidity premium is a measure of the yield that an investor is missing out on if she has to liquidate early. A graphical representation of it is available in figure 7 in appendix E.

Since by lemma 1 the price of capital Q is strictly greater than one, then we have a strictly positive liquidity premium. This corresponds to an upward-sloping yield curve, whereby longer-term assets have higher yields. A strictly positive liquidity premium of capital is a necessary condition for consumers to be exposed to liquidity risk, the risk of having to liquidate their assets early and thus missing out on the liquidity premium.

[10] Yield is conventionally defined as the rate y that sets the net present value of the payoffs π_t of an investment equal to zero:

$$\sum_{t=0}^{T} \frac{\pi_t}{(1+y)^t} = 0 \tag{5}$$

It is a measure of the profitability of an investment.

2.2 Consumers

Consumers make saving and portfolio allocation decisions under idiosyncratic liquidity risk. In this paragraph, I first present the maximisation problem facing consumers and describe consumers' optimality conditions. The last subparagraph discusses in detail the nature of idiosyncratic liquidity risk in this model and its implications.

2.2.1 Consumer's Maximisation Problem

Consider an economy with a unit measure of consumers, who are identical as of time 0. All consumers enjoy consumption at time 0. Then, at time 1 they are subject to a privately-observed liquidity shock θ: with probability $\phi \in (0,1)$ they become early types with $\theta = E$, who enjoy consumption only at time 1; and with probability $1-\phi$ they become late types with $\theta = L$, who enjoy consumption only at time two. The consumer's von Neumann-Morgenstern utility function is therefore

$$U(C_0, C_1, C_2) = u(C_0) + \beta \cdot [\phi \cdot u(C_1) + (1-\phi) \cdot u(C_2)], \tag{7}$$

where C_t denotes consumption of goods at time t and $\beta > 0$ is the consumer's discount factor. I restrict the period utility function to be

$$u(C) = \frac{C^{1-\gamma} - 1}{1-\gamma}, \tag{8}$$

$\gamma > 1$ being the coefficient of relative risk aversion.[11]

At time 0, every consumer receives an endowment of goods Y, profits from banks Π_B and from firms Π_F, and transfers from the central bank T. They can use their disposable income at time 0 to consume C_0 and to purchase assets. Consumers' time-0 decisions are not contingent on their type because the uncertainty is only resolved in time 1. There are two assets that the consumers can buy: physical capital $K_{C,0}$, which earns a per-period net return r, at price Q_0, and deposits D.[12] The consumer's time-0 budget constraint is thus given by

$$C_0 + Q_0 \cdot K_{C,0} + D = Y + \Pi_B + \Pi_F + T. \tag{9}$$

At time 1, the idiosyncratic liquidity shock is realised. Thus, consumer's decisions at time 1 are contingent on their type realisation. They can liquidate L_C units of the physical capital they own or they can purchase new capital $K_{C,1}$ on the primary market at price Q_1, where Q_1 is the price of capital in consumption terms. I rule out a secondary market for physical capital where early-type consumers sell used capital to late types.[13] Consumers

[11] These restrictions on the utility function are standard in the maturity-transformation literature (Diamond and Dybvig, 1983; Farhi et al., 2009).

[12] The assumption that consumers do not hold paper currency is harmless. Consumers would always prefer capital holdings over paper currency, because the short-term return on capital goods cannot fall below a lower bound, given by the return on paper currency, while in the long term capital pays a liquidity premium over paper currency.

[13] Jacklin (1987) shows that transaction costs in the secondary market for capital are necessary for maturity transformation to coexist with capital markets in the model by Diamond and Dybvig (1983). Luck and

can also withdraw W_C from their bank deposit and purchase C_1 consumption goods. The budget constraint at time 1 is then given by

$$C_1 + Q_1 \cdot K_{C,1} = L_C + W_C. \tag{10}$$

At time 2, consumers use their remaining assets to purchase consumption goods C_2 according to

$$C_2 = (1+r) \cdot \Big[(1+r) \cdot K_{C,0} - L_C + K_{C,1}\Big] + (1+d_1) \cdot \Big[(1+d_0) \cdot D_C - W_C\Big]. \tag{11}$$

d_0 and d_1 are the real interest rates on deposits respectively from time 0 to time 1 and from time 1 to time 2.

The formal definition of the consumer's maximisation problem is given by

Definition 2 (Consumer's maximisation problem).
Taking as given prices (Q_0, Q_1, d_0, d_1), bank and firm profits (Π_B, Π_F), and taxes T, the consumer chooses a consumption profile (C_0, C_1, C_2), a time-0 portfolio allocation $(K_{C,0}, D_C)$, time-1 liquidations L_C, purchases of new capital $K_{C,1}$ and deposit withdrawals W_C in order to maximize her expected utility (7) subject to budget constraints (9), (10) and (11), non-negativity constraints on consumption, deposits, time-0 capital purchases, time-1 capital liquidations and purchases

$$(C_0, C_1, C_2, D, K_{C,0}, L_C, K_{C,1}) \geq 0, \tag{12}$$

an upper limit on early deposit withdrawals

$$W_C \leq (1+d_0) \cdot D \tag{13}$$

and capital liquidations

$$L_C \leq (1+r) \cdot K_{C,0}. \tag{14}$$

Decisions made at time 1 and 2, $(C_1, C_2, W_C, L_C, K_{C,1})$, are contingent on the consumer's realisation of θ.

2.2.2 Consumer's Optimality Conditions

At time 0 the consumer decides how much of her wealth to save directly in capital and how much to save in deposits. This optimal portfolio decision is governed by an optimality condition with respect to physical capital $K_{C,0}$ and one by an optimality condition with respect to deposits D.

The optimality condition with respect to capital $K_{C,0}$ is given by

$$C_0^{-\gamma} = \frac{1+r}{Q_0} \cdot \left[\phi \cdot \beta \cdot C_1^{-\gamma} + (1-\phi) \cdot \max\left\{1+r, \frac{1+r}{Q_1}, 1+d_1\right\} \cdot \beta \cdot C_2^{-\gamma}\right] + \xi^K, \tag{15}$$

Schempp (2015) explain that they can be justified by a lemons problem: the average quality of assets on the secondary market is lower than the average quality of new assets because some agents mimic liquidity needs and trade on private information about their assets. Eisfeldt (2004) provides a formal treatment of the combined effect of liquidity shocks and asymmetric information on the outcomes of secondary asset markets.

where $\xi^K \geq 0$ is the Kuhn-Tucker multiplier associated with complementary slackness condition

$$\xi^K \cdot K_{C,0} = 0. \tag{16}$$

At time 1, the consumer's type is realised. If she is an early type, then she uses all her assets for immediate consumption. This implies that the consumer liquidates all her capital. If she is a late type, then she has three options: hold on to the capital, liquidate the capital and purchase new capital at price Q_1, or deposit her capital in a bank and earn net return d_1. The consumer chooses the option that maximises her utility. Given that $Q_1 > 1$, as per lemma 1, it is clear that in equilibrium late types do not liquidate their capital early.

The optimality condition with respect to deposits D is given by

$$C_0^{-\gamma} = (1+d_0) \cdot \left[\phi \cdot \beta \cdot C_1^{-\gamma} + (1-\phi) \cdot \max\left\{1+d_1, \frac{1+r}{Q_1}\right\} \cdot \beta \cdot C_2^{-\gamma}\right] + \xi^D \tag{17}$$

where $\xi^D \geq 0$ is the Kuhn-Tucker multiplier associated with the non-negativity constraint on deposits. The complementary slackness condition is

$$\xi^D \cdot D = 0. \tag{18}$$

If the consumer turns out to be an early type, then at time 1 she withdraws all her deposits and spends them on consumption. If instead she is a late type, then the consumer can either hold on to her deposits or purchase new capital goods at time 1. As according to (17) Equation (17) contains the consumer's optimal decision with respect to time-1 deposit withdrawal W_C. If the consumer is an early type, she withdraws her deposits early and consumes them. If the consumer is a late type, she has two options: hold the deposits until time 2, earning a return $1 + d_1$; or withdraw and reinvest the deposits in capital, earning a return $\frac{1+r}{Q_1}$. Of course, the consumer will take the decision that gives her the highest return. So, as long as $1 + d_1 \geq \frac{1+r}{Q_1}$, late types will hold on to their deposits until time 2.

2.2.3 Liquidity Risk

Consumers are subject to idiosyncratic liquidity risk when they make their time-0 saving and portfolio allocation decisions.

Idiosyncratic liquidity risk is given by the combination of idiosyncratic liquidity shocks, which force some consumers to unwind their investment positions early, and an upward sloping yield curve, which rewards with the liquidity premium those agents who are able to hold on to their investment for longer time. Consumers are at risk of having to liquidate early and thus of missing out on the liquidity premium.

Consumers demand insurance for such risk. In fact, they would benefit from writing an insurance contract contingent on type realisation, whereby consumers who turn out to be late types transfer a part of the liquidity premium that they earned to the early types. However, private observability of the liquidity shock makes this contract not incentive compatible, since consumers would have the incentive to claim that they are early types regardless of their true liquidity needs.

Bank deposits emerge as an incentive-compatible way to provide liquidity-risk insurance.

2.3 Banking Sector

Banks have two roles in the economy. They supply deposits to consumers and, by holding reserves and participating in the interbank market for reserves, they transmit monetary policy to the economy.

Banks provide consumers with liquidity-risk insurance by offering deposit contracts. The essence of the deposit contract is that it gives those consumers hit by a liquidity shock, who therefore withdraw their deposits early, a higher yield than they would have earned by purchasing capital goods and liquidating them early. Banks make sure that late types do not have an incentive to withdraw early too, by offering them between time 1 and time 2 a return at least as high as the return available over the same period in capital markets. Crucially, this return does not include the liquidity premium, as it is a one-period return. Thus, banks can use the liquidity premium that they earn on their long-term assets to insure consumers.

Banks are not subject to liquidity shocks themselves, because depositors' liquidity shocks are idiosyncratic. Thus, by the law of large numbers each banks knows the share of its depositors who are hit by the liquidity shock.

In this paper, I study the implications of maturity transformation for interest-rate setting. I do not study the chief concern of most of the literature on maturity transformation: multiple equilibria, with the presence of the bank-run equilibrium. While bank runs are an interesting feature of models of maturity transformation, their implications for monetary policy are not relevant for the central bank's interest-rate policy, which is my focus. In Diamond and Dybvig (1983) bank runs are sunspot events that can be eliminated if the government provides a credible deposit insurance or if the central bank acts as lender of last resort. So, the absence of the bank-run equilibrium in my model can be thought of as consequence of the central bank perfectly acting as lender of last resort and thus eliminating bank runs.[14]

In the following subparagraph, I describe the bank's maximisation problem. Then, in subparagraph two I shed light on the two-fold role of banks in this model: conduit of monetary policy and provider of liquidity-risk insurance via deposits.

2.3.1 Bank's Maximisation Problem

There is a unit mass of identical banks that maximise their profits Π_B.

Banks offer deposit contracts characterised by interest rates (d_0, d_1), which are respectively the interest rate on deposits from time 0 to time 1 and the rate from time 1 to time 2. The market for deposits features Bertrand competition.

The consumer's optimality conditions define a demand for deposits

$$D = D_C(d_0, d_1; \nu) \tag{19}$$

function of the deposit rates and a vector of variables ν which the bank takes as given. Moreover, banks know the timing of withdrawal of their deposits by using the law of large

[14] My model does not feature a bank-run equilibrium, because banks are not subject to a sequential-service constraint. The sequential-service constraint imposed in Diamond and Dybvig (1983) is the one assumption that gives the individual depositor the incentive to run, because it makes the depositor's payoff depend on her position in line if other depositors run.

numbers to integrate over the individual consumer's optimal timing of withdrawal, given by equation (**??**). The aggregate withdrawal for the bank is given by

$$W = \begin{cases} \phi \cdot (1+d_0) \cdot D & \text{if } 1+d_1 \geq \frac{1+r}{Q_1}, \\ \\ (1+d_0) \cdot D & \text{otherwise.} \end{cases} \tag{20}$$

To invest the resources they collect with deposits, banks have at their disposal three assets: physical capital K_B, bank reserves R and paper money M. Reserves and paper money are 1-period nominal assets supplied by the central bank which are exchangeable one for one. While the former pay a nominal interest rate i^R, the latter do not pay a return but have a real cost of storage $-\frac{LB}{P_1}$. Reserves F can be lent in the interbank market at nominal interest rate i. The bank's investment decision must therefore comply with

$$\Pi_B + Q_0 \cdot K_B + \frac{R+M+F}{P_0} = D \tag{21}$$

I assume a simple demand for bank reserves,

$$\frac{R}{P_0} \geq \rho \cdot D. \tag{22}$$

Banks hold a share of their deposits $\rho > 0$ in bank reserves. The interpretation that I offer for inequality (22) is that reserves play an unmodelled role in the payment system between banks. It is cheaper to settle payments between banks with reserves, which are electronic entries in the central bank's balance sheet, than by transferring paper money. Alternatively, ρ can be interpreted as a reserve requirement imposed by the central bank, in a system where required reserves receive the negative interest rate payment.[15]

In period 1, depositors withdraw W deposits. Each bank meets these payments by liquidating some of its capital L_B and using its money and reserve holdings according to

$$W = L_B + \frac{(1+LB) \cdot M + (1+i^R) \cdot R + (1+i) \cdot F}{P_1}, \tag{23}$$

and late deposit withdrawals are met with the remaining assets of the bank at time 2 according to

$$(1+d_1) \cdot [(1+d_0) \cdot D - W] = (1+r) \cdot [(1+r) \cdot K_B - L_B]. \tag{24}$$

The formal definition of the bank's maximisation problem is given by

Definition 3 (Bank's maximisation problem).
Taking as given prices (Q_0, Q_1, i, i^R), each bank offers a deposit contract (d_0, d_1) and supplies deposits D. It chooses a portfolio of assets at time 0 (K_B, R, M, F), an amount of capital to liquidate at time 1 L_B, an amount of deposit withdrawals W, and profits Π_B. The bank maximises its profits Π_B, given by equations (21), (23) and (24), subject to consumers' demand for deposits (19) and withdrawing behaviour (20), the demand for reserves (22), non-negativity constraints $(K_B, R, M, L_B) \geq 0$ and an upper limit on capital liquidations $L_B \leq (1+r) \cdot K_B$.

[15] In my analysis, I do not differentiate between excess reserves and required reserves. I choose to do this for simplicity. Required rates are exempted from the negative interest on reserves in the current implementations of negative interest rate policies. The mechanism described in my model works as long as there is a strictly positive supply of reserves in the economy to which the negative interest applies.

2.3.2 Banks and the Transmission of Conventional Monetary Policy

One of the roles of the banking system in this model is to be the conduit of conventional monetary policy. Conventional monetary policy is the use of the interest on reserves and of open market operations to control the interbank rate, which is the equilibrium rate that prevails in the interbank market for reserves. By arbitrage, then the interbank rate spills over to the capital market.

The following results are equilibrium results, in that they hinge on the market clearing condition for the interbank market

$$F = 0 \tag{25}$$

holding. Banks hold the amount of reserves and paper money that the central bank supplies with open market operations. In aggregate, banks cannot create more reserves or paper money. Hence, the sum of their borrowings and loans must equal zero. Since banks are identical, they will all symmetrically lend 0 in the interbank market.

The first result is the presence of a lower bound on the interbank rate. Bank behaviour in the interbank market determines the interbank rates that the central bank can achieve. At sufficiently low interbank rates it becomes profitable for all banks to borrow infinitely in the interbank market to hold reserves or store them as paper money. So, these low interbank rate cannot be implemented in the equilibrium.

Lemma 2 (**Lower Bound on the Interbank Rate**).
In equilibrium, it must be that

$$i \geq \max\left\{i^R, LB\right\}. \tag{26}$$

Proof. Consider $i < LB$. If a bank borrowed one unit of reserves in the interbank market, exchanged into paper money and stored, it would make a profit $\Pi_B = \frac{LB-i}{(1+LB)\cdot P_0} > 0$. Therefore, it is optimal for the bank to lend $F = -\infty$, and this is inconsistent with equilibrium requirement (25). □

Another way to phrase lemma 2 is that for any combination of OMO and i^R, it is true that $i \geq \max\left\{i^R, LB\right\}$.

The second result shows how banks' arbitraging behaviour ensures that the real return in the interbank rate is equal to the short-term real return in the capital market.

Lemma 3 (**Arbitrage condition**).
In equilibrium, by arbitrage it must be that

$$(1+i) \cdot \frac{P_0}{P_1} = \frac{1+r}{Q_0}. \tag{27}$$

Proof. Consider $(1+i) \cdot \frac{P_0}{P_1} < \frac{1+r}{Q_0}$. Then, each bank would borrow infinitely in the interbank market to purchase capital. And this is not consistent with market clearing in the interbank market, which requires $F = 0$. The other strict inequality can be similarly proven to be impossible in equilibrium. □

Banks arbitrage between the interbank market and capital markets, and by doing so transmit the monetary stance to capital markets.

2.3.3 Equilibrium Deposit Contract

Banks supply deposit contracts to consumers under Bertrand competition. Deposits are not just a veil that gives consumers access to the returns available in the capital market. Banks carry out maturity transformation, and by doing so they provide liquidity-risk insurance.

By Bertrand competition, banks make zero profits. It is also easy to show that banks will set the interest on deposits at time one high enough that late types do not have an incentive to mimic early types, $1+d_1 \geq \frac{1+r}{Q_1}$. It follows that in equilibrium the following zero-profit condition holds,

$$\phi \cdot (1+r) \cdot (1+d_0) + (1-\phi) \cdot (1+d_1) \cdot (1+d_0) = (1+r) \cdot \left[(1-\rho) \cdot \frac{1+r}{Q_0} + \rho \cdot (1+i^R) \cdot \frac{P_0}{P_1} \right]. \quad (28)$$

The zero-profit condition is a restriction to the equilibrium deposit rates (d_0, d_1). However, it does not pin down the two deposit rates individually.

I find the equilibrium deposit contracts first for the case in which the short-term real return on reserves $(1+i^R) \cdot \frac{P_0}{P_1}$ is equal to the short-term return on capital goods $\frac{1+r}{Q_0}$, and second for the case in which the short-term real return on reserves is lower than that on capital.

Lemma 4 (**Equilibrium deposit contract with slack demand for reserves**).
If $(1+i^R) \cdot \frac{P_0}{P_1} = \frac{1+r}{Q_0}$, *the equilibrium deposit contract features deposit rates*

$$1+d_0 = \frac{\alpha}{\phi \cdot \alpha + 1 - \phi} \cdot \frac{1+r}{Q_0}, \quad (29)$$

$$(1+d_0) \cdot (1+d_1) = \frac{1}{\phi \cdot \alpha + 1 - \phi} \cdot \frac{(1+r)^2}{Q_0}, \quad (30)$$

with

$$\alpha \in \left[(1+r)^{\frac{\gamma-1}{\gamma}}, Q_1 \right]. \quad (31)$$

Proof. Please refer to appendix D. □

There are multiple deposit contracts that are compatible with equilibrium, with each equilibrium deposit contract indexed by a value α belonging to the range in condition (31). Importantly, this multiplicity is irrelevant for the levels and pattern of consumption. In fact, it comes from the fact that the same level of liquidity risk insurance can be achieved by holding fewer deposits with more liquidity-risk insurance per unit of deposit (i.e., a high α within in the range) or by holding more deposits that provide less liquidity-risk insurance per unit (i.e., a low α within in the range). Notice that in all the equilibrium deposit contracts $\alpha > 1$. This means that banks provide liquidity-risk insurance in equilibrium. From equation (29) one can see that the bank gives early withdrawers, who are the consumers hit by the liquidity shock, a higher return than is available on short-term holdings in the capital market. Conversely, equation (30) shows that late withdrawers earn less on their deposits than they would have by directly investing in the capital market at time 0 and then holding the asset for two periods.

If the reserve requirement is binding, then there is a unique deposit contract in equilibrium. Only the deposit contract that provides the maximum incentive-compatible amount of liquidity-risk insurance per unit deposited (i.e., $\alpha = Q_1$) can be sustained, because consumers want to hold as little of their wealth in deposits as possible, given that the binding reserve requirement effectively works as a tax on deposits.

Lemma 5 (**Equilibrium deposit contract with binding demand for reserves**).
If $(1+i^R)\cdot\frac{P_0}{P_1} < \frac{1+r}{Q_0}$*, the equilibrium deposit contract features deposit rates*

$$1+d_0 = \frac{1}{\phi + (1-\phi)\cdot\frac{1}{Q_1}}\cdot\left[(1-\rho)\cdot\frac{1+r}{Q_0} + \rho\cdot(1+i^R)\cdot\frac{P_0}{P_1}\right] \tag{32}$$

$$1+d_1 = \frac{1+r}{Q_1} \tag{33}$$

Proof. Please refer to appendix D. □

2.4 Monetary Policy

The canonical New Keynesian model describes monetary policy in terms of a rule to set the interbank rate, the only interest rate whose macroeconomic effects are of interest. Implicitly, the interbank rate, which is an equilibrium outcome from market clearing in the interbank market, is controlled by means of an interest on reserves and of open market operations that regulate the supply of reserves. Since the central bank controls the interbank rate only indirectly, it is possible that the equilibrium requirements constrain it in terms of the interbank rate that it can achieve. Indeed, this is the essence of the lower bound on the interbank rate. Given that banks can exchange their reserves one for one into paper money, there is no combination of interest on reserves and open market operations that can deliver an interbank rate below the lower bound.

In this model, I am interested in the macroeconomic consequences of multiple interest rates: the interbank rate and the interest on reserves. The central bank directly sets the latter and controls the former by means of open market operations. There is therefore a lower bound on the interbank rate, but no lower bound on the interest on reserves, which the central bank can set by decree to any value.

I model the setting of the interbank rate with a rule,

$$i = \max\left\{i^n, LB\right\}. \tag{34}$$

The rule sets the nominal interest rate equal to the natural real interest rate i^n, which I will define in the next section when discussing the flexible-price equilibrium. The natural real interest rate is the only value of the short-term real interest rate such that the goods market clears. The lower bound $i \geq LB$ must be included to ensure that the Taylor rule complies with lemma 2 and hence with equilibrium.

And lemma 2 imposes another restriction on monetary policy. The setting of the interest on reserve is subject to inequality constraint

$$i^R \leq i. \tag{35}$$

As is shown in section 5, this inequality is never binding for optimal monetary policy, because it would never be welfare-improving to set the interest on reserves higher than the interbank rate.

The central bank must comply with budget constraints. At time 0, the budget constraint is given by

$$\frac{R+M}{P_0} = Q_0 \cdot K_{CB} + T, \tag{36}$$

where T is a lumpsum transfer of the seignourage revenue to consumers. Reserves and money are supplied monopolistically by the central bank via open market operation OMO, so that

$$R + M = OMO. \tag{37}$$

At time 1, the central bank pays off its liabilities with its assets according to

$$\frac{\left(1+i^R\right) \cdot R + M}{P_1} = (1+r) \cdot K_{CB}. \tag{38}$$

2.5 Shocks

The economy is hit by demand shocks in this model, given by changes in the discount factor β. I do not put restrictions on the CDF $F(\beta)$, except that the variance of β is strictly positive.

Demand shocks are the most commonly studied shocks in the context of the modern liquidity-trap literature, starting with Krugman (1998). This is because episodes of extremely low nominal interest rates have taken place in conjunction with below-target inflation and low real GDP growth, suggesting that the lower bound on the interbank rate is most relevant to adverse demand shocks. A demand shock is essentially a change in the natural real interest rate, modelled as a shock to the consumers' time preference. After an adverse demand shocks, consumers will consume enough in the current period only if the reward from saving is sufficiently low, which implies a low real interest rate. In this model, I model this by assuming that the discount factor β is stochastic. A high realisation of β means that, given a real interest rate, consumers want to save more and consume less. Thus, it is an adverse demand shock.

Modelling demand shocks as changes in β is a simplification. A strand of the liquidity-trap literature (Guerrieri and Lorenzoni, 2011; Eggertsson and Krugman, 2012) puts more structure on the demand shock. At the cost of a more complex model, they show that a reduction in the natural real interest rate can be the result of a tightening in the borrowing constraint on agents with a higher propensity to consume. In this sense, an adverse demand shock can be thought of as a financial crisis.

3 Equilibrium

In this section of the paper, I define two equilibrium concepts. First, I define the competitive equilibrium with flexible prices. This allows me to find the real interest rate that must prevail for markets to clear: the natural real rate of interest. Second, I define an equilibrium concept with sticky prices. In this equilibrium, I can meaningfully study monetary policy, which has real effects. Demand and supply are not automatically equated by changes in the price level. Hence, it is the central bank's task to manage demand in order to ensure the full employment of the endowment. I will assume that the central bank is benevolent and knows exactly the interest rate that ensures market clearing. However, if monetary policy is constrained by the lower bound, then the equilibrium features rationing in the time-0 goods market.

3.1 Flexible-Price Equilibrium and the Natural Real Interest Rate

In the flexible-price equilibrium, firms, banks and consumers solve their optimisation problems and prices adjust to ensure market clearing. The equilibrium features full employment of the endowment and monetary policy has no real effects.

Please refer to definition 7 in appendix A for a formalisation of the flexible-price equilibrium.

The flexible-price equilibrium is useful, because it features the short-term real interest rate which, conditional on the shock β and on the interest on reserves i^R, is necessary for all markets to clear. This is the natural real rate of interest.

Definition 4 (Natural real rate of interest).
Define the natural real rate of interest i^n as equal to the short-term real return on capital in the flexible-price equilibrium,

$$1 + i^n \equiv \frac{1+r}{Q_0^{FP}}. \tag{39}$$

3.2 Sticky-Price Equilibrium

I define an equilibrium with sticky prices in order to study optimal monetary policy, because nominal rigidities are necessary for monetary policy to have real effects.

I assume a simplified notion of nominal rigidity. In the short run, prices are fixed with $P_1 = P_0 = 1$.

Firms, banks and consumers solve their optimisation problems, and monetary policy sets the interbank rate according to rule (34) and the interest on reserves $i^R \leq i$.

Please refer to definition 8 in appendix A for a formalisation of the sticky-price equilibrium.

Since prices cannot adjust to ensure market clearing, there is rationing in the time-0 goods market, depending on the shock and on monetary policy. Rationing takes place when at the going price there is not enough demand to absorb all the goods supplied, because the return on saving is too high.[16] The role of monetary policy is to adjust its interest rates in order to discourage consumers from saving excessively, because this leads to rationing. However, the presence of a lower bound on the interbank rate may make it impossible for the central bank to offset large adverse demand shocks. In these cases, which I call liquidity traps, rationing takes place in equilibrium.

In the following subparagraph, I show the system of equations that determines the sticky-price equilibrium allocation. Then, I define the liquidity trap in this model and discuss the relationship between liquidity trap and rationing.

3.2.1 Equilibrium allocation

Given an interest rate on reserves i^R, variables $(C_0, C_1, C_2, K_0, X, Q_0, i)$ are determined in the sticky-price equilibrium by the following conditions.

First, I define an index of liquidity-risk insurance

$$X \equiv \phi + (1-\phi)\cdot(1+r)\cdot\left(\frac{C_2}{C_1}\right)^{-\gamma}. \tag{40}$$

If the marginal rate of substitution between time 1 and time 2, $\left(\frac{C_2}{C_1}\right)^{\gamma}$, is equal to the marginal rate of transformation $1+r$, then there is perfect liquidity-risk insurance and $X = 1$. $X < 1$ indicates imperfect liquidity-risk insurance in the economy.

Thus defined, the index of liquidity-risk insurance plays an important role in the equilibrium allocation in that it appears in the Euler equation

$$C_0^{-\gamma} = (1+i)\cdot\beta\cdot C_1^{-\gamma}\cdot X. \tag{41}$$

A higher index of liquidity-risk insurance makes it more desirable to save at time 0, as it is less risky. The Euler equation is derived from the consumer's demand for capital (15) combined with the definition of the index of liquidity-risk insurance (40).[17]

The equilibrium level of liquidity-risk insurance is determined by the consumers' portfolio decision between deposits and direct holdings of capital. And this portfolio decision hinges on i^R. Reserves being paid a lower interest than is available in the capital market is equivalent to a tax on bank deposits. It follows that consumers will decrease their holdings of deposits in favour of direct holdings of capital. As a consequence, liquidity-risk insurance will be less than perfect.

$$X = \max\left\{\frac{1}{1+\frac{\lambda\cdot\phi\cdot\rho}{1-\lambda}\cdot\frac{i-i^R}{1+i}}, \hat{X}\right\}, \tag{42}$$

where $\hat{X} = \phi + (1-\phi)\cdot(1+r)^{1-\gamma} < 1$ is the level of liquidity risk-insurance that prevails when deposits are shunned completely on account of an excessively low interest on bank

[16] Buyers are rationed. Sellers do not consume the part of their endowment that they are unable to sell, because by assumption consumers do not like their own endowment and thus trade is necessary.

[17] $\xi^K = 0$, as follows immediately from lemmas 4 and 5.

reserves.[18] $\hat{X}$ is therefore the minimum level of liquidity-risk insurance that can be attained in equilibrium by low interest on reserves.[19]

The interest rate in the interbank market is set by the central bank according to Taylor rule (34), which is constrained by a lower bound. Given that in equilibrium the price-level is determined at the target P^T, the interbank-rate setting rule can be written as

$$i = \max\Big\{i^n, LB\Big\}. \tag{34}$$

The central bank knows the natural rate that sets demand equal to supply and therefore avoids any rationing, i^n. It sets the interbank rate equal to it, unless i^n falls below the lower bound LB. In such case, monetary policy remains excessively tight.

The interbank rate spills over to the capital market by arbitrage, as shown in lemma 3. In equilibrium, the short-term return on capital is equal to the interbank interest rate,

$$\frac{1+r}{Q_0} = 1+i \tag{43}$$

This equation pins down the equilibrium price of capital. Then, using the capital supply function

$$K_0 = K^S(Q_0), \tag{4}$$

we can back out the equilibrium level of capital.

Market clearing in the market for consumption goods takes place at time 1 and at time 2. Thereby,

$$(1-\phi)\cdot C_2 = (1+r)\cdot\Big[(1+r)\cdot K_0 - \phi\cdot C_1\Big]. \tag{44}$$

Lemma 6 (**Sticky-price equilibrium allocation**).
In the sticky-price equilibrium in definition 8, variables $(C_0, C_1, C_2, K_0, X, Q_0, i)$ are uniquely determined as a function of the shock β and monetary policy i^R by equations (40), (41), (42),(34), (43), (4) and (44).

3.2.2 Rationing

The equilibrium features two regimes: a full-employment regime and a rationing regime, according to whether condition

$$C_0 \leq Y - f^{-1}(K_0). \tag{45}$$

holds with equality or not. Because of the nominal rigidity, prices do not adjust to ensure market clearing. Hence, it is the role of conventional monetary policy to manage aggregate demand in a way that makes the economy fully utilise the endowment.

The central bank sets the interbank rate to ensure full utilisation of the endowment. By definition of the natural real rate of interest i^n, this is what the central bank is doing when it sets $i = i^n$, according to its interest-setting rule (34). However, if $i^n < LB$, then it becomes impossible for the central bank to pursue this objective and rationing occurs.

[18] Expression (42) can be derived by combining the consumer's demand for capital (15) and for deposits (17).

[19] I derive $\hat{X}$ in section C, where I discuss the bank-less equilibrium allocation.

Lemma 7 (**Rationing in the sticky-price equilibrium**).
In the sticky-price equilibrium, we have rationing with

$$C_0 < Y - f^{-1}(K_0) \tag{46}$$

if and only if

$$i^n\Big(\beta, i^R\Big) < LB. \tag{47}$$

Proof. This follows from the definition of the natural real rate of interest $i^n\Big(\beta, i^R\Big)$ in 4 and from the setting rule for the interbank rate, equation (34). Given that time-0 consumption and investment demand are strictly decreasing in the interbank rate, as can be seen from respectively equation (41) and equation (4), an interbank rate strictly larger than i^n leads to rationing. □

4 Social Planner

In this section, I study the conditions for efficiency of an allocation in the economy that I described thus far. I do so by solving the social planner's problem.

I use a strong notion of social planner, which is only subject to the economy's resource contraints, given by

$$C_0 \leq Y - f^{-1}(K_0), \tag{48}$$

$$\phi \cdot C_1 + f^{-1}(K_1) \leq L, \tag{49}$$

$$(1-\phi) \cdot C_2 \leq (1+r) \cdot \Big[(1+r) \cdot K_0 - L + K_1\Big]. \tag{50}$$

The model's informational friction does not constrain the social planner: it can observe the consumers' types and allocate consumption accordingly.

Definition 5 (**Social planner's problem**).
The social planner maximises aggregate welfare, given by function (7), subject to constraints (48), (49) and (50) with respect to variables $(C_0, C_1, C_2, K_0, L, K_1)$.

Solving the social planner's problem determines the first-best allocation $(C_0^*, C_1^*, C_2^*, K_0^*)$, given by equations

$$C_0^{*^{-\gamma}} = (1+r) \cdot f'\Big[f^{-1}(K_0^*)\Big] \cdot \beta \cdot C_1^{*^{-\gamma}}, \tag{51}$$

$$\left(\frac{C_2^*}{C_1^*}\right)^{\gamma} = 1 + r, \tag{52}$$

$$C_0^* + f^{-1}(K_0^*) = Y, \tag{53}$$

$$(1-\phi) \cdot C_2^* = (1+r) \cdot \Big[(1+r) \cdot K_0^* - \phi \cdot C_1^*\Big]. \tag{54}$$

Proposition 1 (**First-best allocation**).
The first-best allocation $(C_0^*, C_1^*, C_2^*, K_0^*)$ *is given by equations (51), (52), (53) and (54).*

Corollary 1 (**Characteristics of first-best allocation**).
If an allocation is first-best, then it is characterised by

1. *Full employment of the endowment, as per condition (53); and*

2. *Perfect liquidity-risk insurance,*
$$X^* = 1. \tag{55}$$

Since consumers value liquidity-risk insurance, the first-best allocation features perfect liquidity risk insurance. Moreover, since utility is strictly increasing in consumption, the first-best allocation features full employment of the endowment. Notably, there is no tradeoff between full employment and full liquidity-risk insurance.

In appendix C, I study another benchmark case of the economy: the disintermediated economy, in which deposits are ruled out.[20] The social planner and the disintermediated benchmark cases are relevant to understanding the economy's reaction to changes in the interest on reserves. In the absence of rationing, the social planner's allocation and the disintermediated allocation are the two extremes of a continuum of allocations that the central bank can implement by setting the interest on reserves.

5 Optimal Monetary Policy

In this section, I study the central bank's optimal monetary policy with respect to the choice of the interest on reserves i^R in the sticky-price equilibrium.

This optimal monetary policy exercise is a Ramsey problem in which the central bank chooses the interest on reserves. The conventional interest rate i is set by rule (34), which ensures full utilisation of the endowment unless the lower bound is binding and is optimal given an interest on reserves.

Definition 6 (**Optimal monetary policy problem**).
The optimal monetary policy problem maximises aggregate welfare, defined in equation (7), subject to the sticky-price equilibrium conditions, defined in lemma 6, by choosing $i^R \leq i$. The maximiser of the problem is a function of the demand shock, $i^R = i^R(\beta)$.

In the first paragraph, I discuss the case in which the central bank can implement the first-best allocation. Then, I study the optimal setting of the interest on reserves when the first-best cannot be achieved. In the third paragraph, I give a definition of liquidity trap and show that the second-best allocation takes place in equilibrium if and only if the economy is in the liquidity trap.

5.1 Implementation of the First-Best Allocation

By definition, a benevolent central bank cannot do better than implementing the first-best allocation. In this paragraph, I study the restrictions to the shock under which the central bank is able to achieve full efficiency, and the optimal interest on reserves

[20] This is also known as the autarky equilibrium in the maturity-transformation literature.

in such case. First, I find the restriction both on β and on i^R such that the equilibrium allocation is first best. Then, I check that there exists a rule to set the interest on reserves such that the restrictions hold for any realisation of the demand shock.

The requirements for the allocation to be first-best, as described in proposition 1, are full employment of the endowment and perfect liquidity-risk insurance. Full employment takes place whenever the lower bound is not in the way of setting the interbank rate at the natural level i^n, as shown in lemma 7. That is, full employment takes place in the equilibrium whenever $i^n\left(\beta, i^R\right) \geq LB$. The equilibrium level of liquidity-risk insurance depends on the interest on reserves, according to condition

$$X = \max\left\{\frac{1}{1 + \frac{\lambda\cdot\phi\cdot\rho}{1-\lambda}\cdot\frac{i-i^R}{1+i}}, \hat{X}\right\}, \tag{42}$$

with $\hat{X} = \phi + (1-\phi)\cdot(1+r)^{1-\gamma} < 1$. This shows that the only level of the interest on reserves that is compatible with perfect liquidity-risk insurance is $i^R = i$. This can be interpreted as a Friedman-rule result. By means of maturity transformation banks provide liquidity-risk insurance to consumers that hold deposits. Since maturity transformation is costless, satiation in deposits is welfare-maximising. An interest on bank reserves lower than the short-term return in the capital market acts as a tax on deposits and incentivises consumers to move their wealth away from deposits. This hurts aggregate welfare by decreasing the equilibrium level of liquidity-risk insurance.

Lemma 8 (**Conditions for implementation of first-best allocation**).
The sticky-price equilibrium allocation is first-best if and only if

1. $i^n\left(\beta, i^R\right) \geq LB$, *and*

2. $i^R = i$.

Does a function $i^R(\beta)$ exist such that the conditions of lemma 8 hold for any β? I find that the first-best allocation is feasible if and only if demand shocks are not too negative. Given a central bank that follows the Friedman rule to defend maturity transformation, large enough adverse demand shocks push the natural interest rate below the lower bound.

Proposition 2 (**Feasibility of implementation of first-best allocation**).
There exists a threshold $\overline{\beta}$ such that

1. *If the demand shock $\beta \leq \overline{\beta}$, then $i^R = i$ implements the first-best allocation; and*

2. *If the demand shock $\beta > \overline{\beta}$, then there exists no $i^R(\beta)$ such that the sticky-price equilibrium allocation is first-best. I call this the liquidity trap.*

Proof. Please refer to appendix D. □

I am defining as liquidity trap the states of the world in which demand shocks are sufficiently large to make the first-best allocation not implementable. So far I have not shown that these states of the world possess the features that are typically associated with the liquidity trap, for example an interbank rate constrained by the lower bound. I cannot do so in this section, because the characteristics of liquidity-trap allocations depend on the setting of the interest on reserves that is optimal in such cases.

Friedman-rule results are ubiquitous in monetary theory, starting with Friedman (1969). Recently, Cúrdia and Woodford (2011) and Cochrane (2014) concluded that the interest on reserves should be set equal to the interbank rate not to unduly damage the banking system, regardless of shocks. The literature, however, mostly focuses on the asset side of banks. It does not consider the role that banks and maturity transformation play in promoting saving behaviour by consumers. In the next paragraph, I study whether inducing disintermediation by deviating from the Friedman rule is optimal when the economy is hit by a large adverse demand shock.

5.2 Liquidity Trap

In this paragraph, I study the optimal setting of the interest on reserves in the liquidity trap. In this case (i.e., $\beta > \overline{\beta}$), a large adverse demand shock hit the economy and therefore the first-best allocation cannot be implemented. Perfect liquidity-risk insurance and full employment are not compatible, as shown in proposition 2.

First of all, notice that if the central bank sticks to the floor system of monetary policy, it preserves a fully functional banking sector, which provides perfect liquidity-risk insurance. However, under such policy there is rationing in the goods market.

If instead it lowers the interest on reserves below the interbank rate, the central bank reduces the level of liquidity-risk insurance by equation

$$X = \max\left\{\frac{1}{1 + \frac{\lambda\cdot\phi\cdot\rho}{1-\lambda}\cdot\frac{i-i^R}{1+i}}, \hat{X}\right\}, \tag{42}$$

where $\hat{X} < 1$. Per se, this represents a deviation from first best. But, liquidity-risk insurance plays a role in the saving decision of the consumer. As can be seen from the Euler equation

$$C_0^{-\gamma} = (1+i)\cdot\beta\cdot C_1^{-\gamma}\cdot X, \tag{41}$$

a reduction in X from $X = 1$ lowers the consumer's incentive to save and therefore boosts her current spending.

So, when the economy is in the liquidity trap, there is a policy tradeoff between preserving a fully effective banking sector and attempting to stimulate the economy with a lower interest on reserves. We can think of the tradeoff as between more current consumption C_0 or more liquidity-risk insurance X, and the policy instrument that strikes the balance is the interest on reserves i^R.

To move forward with the analysis, we can make a first statement on optimal monetary policy in the liquidity trap.

Lemma 9 (**Binding lower bound in second-best allocation**).
If $\beta > \overline{\beta}$ as defined in proposition 2, then $i^n\left[\beta, i^R(\beta)\right] \leq LB$. This implies that $i = LB$.

The intuition is that varying the interbank rate is the most effective way for the central bank to respond to demand shocks. It follows that it is suboptimal for the central bank to ease so much by means of the interest on reserves that the economy needs to be cooled down with the interbank rate. This lemma tells us that in the second-best allocation the lower bound on the interbank rate is always binding.

By bank arbitrage (3), the interbank rate determines a price of capital. Thus, in this case the price of capital is given by

$$Q_0 = \frac{1+r}{1+LB}. \tag{56}$$

Q_0 maps into a quantity of capital K_0 through the capital supply function (4). With a given interbank rate, lowering the interest on reserves does not affect capital accumulation in the economy, because banks and consumers invest in capital up to the point in which the short-term return on capital is lower than the short-term return on interbank lending.[21] Given the level of capital, the resource constraint (44) tells us how much consumption the consumers will enjoy on average. The distribution of consumption between the types is given by the equilibrium level of X, defined by (40). Equations (56), (4), (44) and (40) determine C_1 and C_2 solely as functions of X, without reference to current spending C_0. Usually current consumption affects future consumption by reducing the amount of resources invested. In this case, there is rationing in the goods market. Thus, an increase in current spending reduces the amount of resources wasted and does not affect the resources available for consumption in future periods.

The finding that the level of capital is pinned down by the lower bound on the interbank rate allows us to greatly simplify the Ramsey problem. The central bank chooses C_0 and X to maximise aggregate welfare

$$\frac{C_0^{1-\gamma}-1}{1-\gamma} + \beta \cdot \left\{ \phi \cdot \frac{\left[C_1(X)\right]^{1-\gamma}-1}{1-\gamma} + (1-\phi) \cdot \frac{\left[C_2(X)\right]^{1-\gamma}-1}{1-\gamma} \right\}, \tag{57}$$

subject to the implementability constraint

$$C_0^{-\gamma} = \beta \cdot (1+LB) \cdot \left[C_1(X)\right]^{-\gamma} \cdot X, \tag{58}$$

and inequality constraints

$$C_0 \leq Y - f^{-1}\left[K^S\left(\frac{1+r}{1+LB}\right)\right], \tag{59}$$

$$X \in \left[\hat{X}, 1\right]. \tag{60}$$

The implementability constraint represents the consumers' optimal saving behaviour, which constrains the allocations that the central bank can implement by means of the interest on reserves.[22]

[21] It does not matter that consumers do not have direct access to the interbank market, because arbitrage by banks transmit the interbank rate to the capital market.

[22] It is the Euler equation (41) combined with the function that determines future consumption in the liquidity trap for early types.

The optimal interest on reserves can be backed out from the optimal level of liquidity-risk insurance using equation

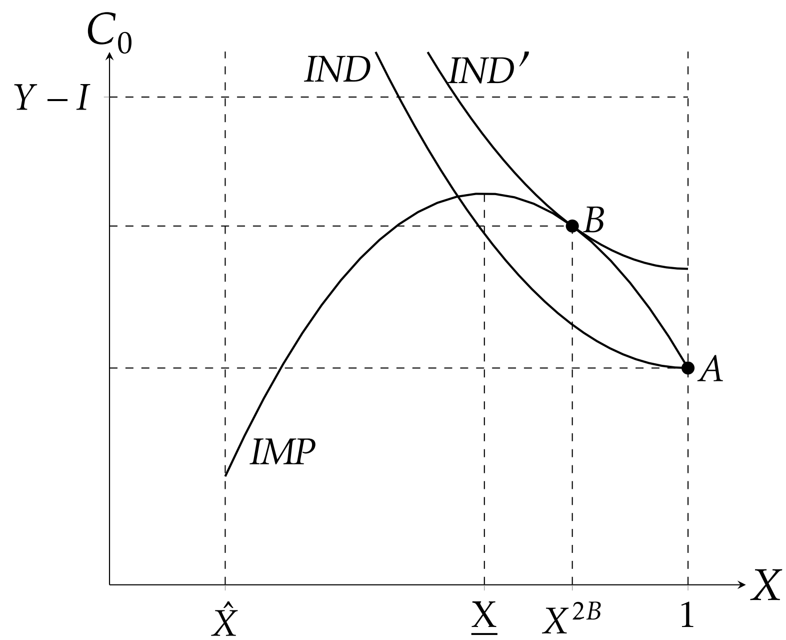
$$X = \max\left\{\frac{1}{1 + \frac{\lambda\cdot\phi\cdot\rho}{1-\lambda}\cdot\frac{LB-i^R}{1+LB}}, \hat{X}\right\}, \tag{61}$$

with $\hat{X} = \phi + (1-\phi)\cdot(1+r)^{1-\gamma} < 1$.

The second-best problem that the central bank faces in the liquidity trap is represented graphically in the (X, C_0) space in figure 3.

Aggregate welfare is represented on the Cartesian plane by a family of indifference curves, labeled *IND*. Aggregate welfare is increasing with current consumption and with liquidity-risk insurance within the range $[\hat{X}, 1]$. At $X = 1$, consumers are satiated in liquidity-risk insurance. This means that, when there is perfect liquidity-risk insurance, consumers are willing to reduce it marginally in exchange for any strictly positive increase in current consumption, however small. This is captured in the figure by a flat indifference curve at $X = 1$.

Figure 3: The second-best problem



The implementability constraint is represented in figure 3 by a curve labeled *IMP*. Point A is the combination of liquidity-risk insurance and time-0 consumption that prevails if the central bank sets $i^R = LB$ and thus implements perfect liquidity-risk insurance. The economy moves along the *IMP* constraint to the left as the interest on reserves is cut. Lower interest on reserves reduces the attractiveness of deposits, as banks are effectively taxed. Hence, consumers substitute their wealth away from deposits into direct capital holdings. However, deposits and capital are not perfect substitutes, in that the former provides liquidity-risk insurance. It follows that the consumer partially substitutes away from overall saving and increases her current consumption. The quasi concavity of the *IMP* curve is given by an income effect, which pulls in the other direction. Consumers are made poorer by the reduction in the attractiveness of investment opportunities. This increases the value of saving. At the point where the implementability curve peaks, the income effect and the substitution effect perfectly cancel each other out. To the left of the peak, the income effect dominates. However, the range of liquidity-risk insurance where the income effect dominates is not important for the optimal monetary policy exercise, because it is always suboptimal for monetary policy to move the economy in it.

Optimality requires that the rate at which consumers would trade off liquidity-risk insurance for more consumption remaining equally well off be equal to the rate at which the central bank can increase consumption by decreasing liquidity-risk insurance. This is given by the tangency point of curve *IND* and *IMP*, point B in figure 3.

Proposition 3 (**Optimal Monetary Policy in Liquidity Trap**).
If the economy is in the liquidity trap as defined in proposition (2), the optimal interest rate on reserves sets $i^R < LB$. It implements a second-best allocation with partial liquidity-risk insurance $X \in (\hat{X}, 1)$.

Point B in figure 3 always features partial liquidity-risk insurance. So, setting the interest on reserves strictly below the lower bound on the interbank rate is optimal. Notice that this monetary policy does not transmit to the economy through the conventional intertemporal substitution channel via a reduction in the real interest rate. It solely transmits by making deposits less attractive for consumers, via the maturity-transformation channel of monetary policy transmission.

The intuition for the result is that a marginal decrease in liquidity-risk insurance from $X = 1$ does not hurt welfare, because the consumer is satiated in X, while it encourages non-zero increases in consumption C_0. The increase in consumption is unambiguosly desirable because it increases the amount of endowment that is utilised. Therefore, it is optimal for the central bank to lower the interest rate on reserves below zero when the economy is experiencing a liquidity trap. The model gives an interior solution for the degree of liquidity-risk insurance because liquidity risk becomes more valuable to consumers as it is reduced and because reductions in liquidity are less and less effective at stimulating consumption.

6 Conclusion

In this paper, I develop a monetary model with two important characteristics: multiple interest rates, which monetary policy controls, and a meaningful banking sector. With reference to the former, I explicitly model the interbank rate, which is the conventional instrument of monetary policy, and the interest on reserves separately. And as regards the latter, banks are modelled as maturity transformers, in accordance with the framework of Diamond and Dybvig (1983).

I have four main findings in the paper. First, I show that the lower bound does not apply to the interest on reserves. A first-pass explanation for this is that the central bank can decide by decree to pay whatever interest on bank reserves, while the interbank rate is the equilibrium price that clears the interbank market. The lower bound emerges as equilibrium requirement on the latter because of the presence of currency. Nonetheless, if reserves and other assets are perfect substitutes as in the canonical New Keynesian model, changing the interest on reserves per se has no macroeconomic effect.

Second, I show that reserves and other assets become imperfect substitutes if banks perform maturity transformation à la Diamond and Dybvig (1983). From the consumer's perspective, deposits and other assets are not perfect substitutes because of the benefits of maturity transformation. Since reserves are necessary to supply redeemable deposits, bank reserves also become imperfect substitutes of other assets. This creates the maturity-transformation channel of monetary policy transmission, whereby a reduction in the interest on reserves, which leaves the interbank rate unchanged, increases aggregate demand.

Third, I find that exploiting the maturity-transformation channel to boost aggregate demand involves an interesting trade-off with the preservation of a fully functional banking system. A lower interest on reserves transmits to the deposit rate. Thus, consumers respond by moving their wealth out of deposits into direct asset holdings. Such disintermediation is detrimental to welfare in this setting, because deposits provide valuable liquidity-risk insurance. Stimulus can be provided to the economy via the maturity-transformation channel only against the backdrop of a weakening banking system.

The last and most important finding of the paper is that, when the interbank rate is constrained by its lower bound, optimal monetary policy prescribes an interest on reserves strictly below the lower bound on the interbank rate. In other words, I find that, when demand is insufficient, there is always some space to stimulate the economy by cutting the interest on reserves below the interest rate prevailing in the money market without damaging the banking sector excessively.

In reality, central banks implement negative interest rate policies by cutting the interest on reserves without knowing where the lower bound on the interbank rate is. They do not know when the cuts to the interest on reserves will stop transmitting to the money market. A discussion has developed over the extent to which central banks should therefore be prudent in lowering their interest on reserves (Dell'Ariccia et al., 2017). The result of my paper can be read as policy recommendation not to be prudent, because lowering the interest on reserves beyond the lower bound on the interbank rate is the optimal monetary policy.

Moreover, my theory contributes a relevant and measurable indicator for the health of the banking sector, which could guide monetary authorities in the implementation of negative interest rates. The degree of liquidity-risk insurance, which represents the effectiveness of the banking sector at providing liquidity, is expressed in terms of observables and therefore can in principle be measured. The theory predicts that negative interest rates lead to a decline in liquidity-risk insurance, and makes clear that, while a moderate reduction is necessary in order to stimulate the economy, an excessive reduction is undesirable. Therefore, monitoring this measure would allow to adjust interest-rate setting in accordance with the level of stress that it imposes on the banking system.

This paper is the first to study the role of the interest rate on bank reserves in the liquidity trap of a model with maturity-transforming banks. Maturity transformation is an important aspect of banking. However, it is by no means the only one. Future research should enrich the modelling of the banking sector, on the asset side (Brunnermeier and Koby, 2017; von Thadden, 1997) and on the liability side with capital constraints. This will let us establish quantitatively the extent to which bank characteristics matter for the effectiveness of negative interest rates. There is also a more general discussion over the role that the banking system plays in depressions: whether weaker banks contribute to the contraction by decreasing aggregate demand (Friedman and Schwartz, 1963) or whether to the contrary weaker banks increase demand by reducing saving opportunities for consumers, as conjectured in Krugman (1998). Pursuing this line of research with richer models could achieve answers to this overarching question, which has many important implications in addition to the optimal level of negative rates: for example, the appropriateness of using public funds to rescue undercapitalised banks, or the optimal cyclicality of capital requirements on banks.

References

Allen, Franklin, and Douglas Gale. 2004. "Financial Intermediaries and Markets." *Econometrica*, 72(4): 1023–1061.

Alsterlind, Jan, Hanna Armelius, David Forsman, Björn Jönsson, and Anna-Lena Wretman. 2015. "How Far Can the Repo Rate Be Cut?." Sveriges Riksbank, URL: http://www.riksbank.se/Documents/Rapporter/Ekonomiska_kommentarer/2015/rap_ek_kom_nr11_150929_eng.pdf.

Basten, Christoph, and Mike Mariathasan. 2017. "Bank Profitability And Risk-Taking When Interest Rates Are Negative."

Bech, Morten Linnemann, and Aytek Malkhozov. 2016. "How Have Central Banks Implemented Negative Policy Rates?." *BIS Quarterly Review*.

Bernanke, Ben. 2016. "What Tools Does the Fed Have Left? Part 1." Brookings Institute, URL: https://www.brookings.edu/blog/ben-bernanke/2016/03/18/what-tools-does-the-fed-have-left-part-1-negative-interest-rates/.

Brunnermeier, Markus, and Yann Koby. 2017. "The Reversal Interest Rate: The Effective Lower Bound of Monetary Policy." Unpublished manuscript.

Burke, Chris, Spence Hilton, Ruth Judson, Kurt Lewis, and David Skeie. 2010. "Reducing the IOER Rate: an Analysis of Options." Federal Reserve, URL: https://www.federalreserve.gov/monetarypolicy/files/FOMC20100805memo05.pdf.

Carvalho, Carlos, Andrea Ferrero, and Fernanda Nechio. 2016. "Demographics and Real Interest Rates: Inspecting the Mechanism." Working Paper Series 2016-5, Federal Reserve Bank of San Francisco.

Clouse, James, Dale Henderson, Athanasios Orphanides, David H. Small, and P.A. Tinsley. 2003. "Monetary Policy When the Nominal Short-Term Interest Rate is Zero." *The B.E. Journal of Macroeconomics*, 3(1): 1–65.

Cochrane, John H. 2014. "Monetary Policy with Interest on Reserves." *Journal of Economic Dynamics and Control*, 49(C): 74–108.

Cúrdia, Vasco, and Michael Woodford. 2011. "The Central-Bank Balance Sheet as an Instrument of Monetary Policy." *Journal of Monetary Economics*, 58(1): 54–79.

Dell'Ariccia, Giovanni, Vikram Haksar, and Tommaso Mancini-Griffoli. 2017. "Negative Interest Rate Policies – Initial Experiences and Assessments." International Monetary Fund.

Diamond, Douglas W. 1997. "Liquidity, Banks, and Markets." *Journal of Political Economy*, 105(5): 928–956.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

Eggertsson, Gauti B., Ragnar E. Juelsrud, and Ella Getz Wold. 2017. "Are Negative Nominal Interest Rates Expansionary?" Working Paper 24039, National Bureau of Economic Research.

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach." *The Quarterly Journal of Economics*, 127(3): 1469–1513.

Eggertsson, Gauti B., and Michael Woodford. 2003. "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 34(1): 139–235.

Eisfeldt, Andrea L. 2004. "Endogenous Liquidity in Asset Markets." *Journal of Finance*, 59(1): 1–30.

Ennis, Huberto M. 2014. "A Simple General Equilibrium Model of Large Excess Reserves." Working Paper 14-14, Federal Reserve Bank of Richmond.

Farhi, Emmanuel, Mikhail Golosov, and Aleh Tsyvinski. 2009. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies*, 76(3): 973–992.

Friedman, Milton. 1969. "The Optimum Quantity of Money."

Friedman, Milton, and Anna J. Schwartz. 1963. *A Monetary History of the United States, 1867–1960*. In , NBER Books(frie63-1): : National Bureau of Economic Research, Inc.

Galí, Jordi. 2008. "Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework." In *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*.: Princeton University Press.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.

Guerrieri, Veronica, and Guido Lorenzoni. 2011. "Credit Crises, Precautionary Savings, and the Liquidity Trap."Technical report, National Bureau of Economic Research, Inc.

Heider, Florian, Farzad Saidi, and Glenn Schepens. 2017. "Life Below Zero: Bank Lending Under Negative Policy Rates." September.

Hellwig, Martin. 1994. "Liquidity Provision, Banking, and the Allocation of Interest Rate Risk." *European Economic Review*, 38(7): 1363–1389.

Hicks, John R. 1937. "Mr. Keynes and the "Classics"; a Suggested Interpretation." *Econometrica*, 5(2): 147–159.

Jacklin, Charles J. 1987. "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade*. eds. by Edward C. Prescott, and Neil Wallace: University of Minnesota Press, 26–47.

Karabarbounis, Loukas, and Brent Neiman. 2013. "The Global Decline of the Labor Share." NBER Working Papers 19136, National Bureau of Economic Research, Inc.

Kashyap, Anil K., and Jeremy C. Stein. 2012. "The Optimal Conduct of Monetary Policy with Interest on Reserves." *American Economic Journal: Macroeconomics*, 4(1): 266–282.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. "The Aggregate Demand for Treasury Debt." *Journal of Political Economy*, 120 233–267.

Krugman, Paul R. 1998. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity*, 29(2): 137–206.

Luck, Stephan, and Paul Schempp. 2015. "Banks, Shadow Banking, and Fragility." Annual Conference 2015 (Muenster): Economic Development - Theory and Policy 113204, Verein für Socialpolitik / German Economic Association.

Reis, Ricardo. 2016. "Funding Quantitative Easing to Target Inflation." Discussion Papers 1626, Centre for Macroeconomics (CFM).

Rognlie, Matthew. 2016. "What Lower Bound? Monetary Policy with Negative Interest Rates." Unpublished manuscript.

Rogoff, Kenneth. 2017. "Dealing with Monetary Paralysis at the Zero Bound." *Journal of Economic Perspectives*, 31(3): 47–66.

Summers, Lawrence H. 2014. "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound." *Business Economics*, 49(2): 65–73.

von Thadden, Ernst-Ludwig. 1997. "The Term-Structure of Investment and the Banks' Insurance Function." *European Economic Review*, 41(7): 1355–1374.

von Thadden, Ernst-Ludwig. 1998. "Intermediated versus Direct Investment: Optimal Liquidity Provision and Dynamic Incentive Compatibility." *Journal of Financial Intermediation*, 7(2): 177–197.

Vayanos, Dimitri, and Jean-Luc Vila. 2009. "A Preferred-Habitat Model of the Term Structure of Interest Rates." CEPR Discussion Papers 7547, CEPR.

Viñals, Jose, Simon Gray, and Kelly Eckhold. 2016. "The Broader View: the Positive Effects of Negative Nominal Interest Rates." International Monetary Fund, URL: `https://blogs.imf.org/2016/04/10/the-broader-view-the-positive-effects-of-negative-nominal-interest-rates`.

Werning, Ivan. 2011. "Managing a Liquidity Trap: Monetary and Fiscal Policy." nber working papers, National Bureau of Economic Research, Inc.

Woodford, Michael. 2003. *Interest and prices*. Princeton, NJ: Princeton Univ. Press.

A Definitions

Definition 7 (Flexible-price equilibrium).
The flexible-price equilibrium consists of quantities

$$(C_0, C_1, C_2, K_{C,0}, K_B, K_{CB}, K_{C,1}, I_0, I_1, L_C, L_B, D, M, R, F, OMO, \Pi_B, \Pi_F, T)$$

and prices

$$(P_0, P_1, Q_0, Q_1, d_0, d_1, i^R, i)$$

such that

1. *Firms solve their optimisation problem as in definition 1.*
2. *Consumers solve their optimisation problem as in definition 2.*
3. *Banks solve their optimisation problem as in definition 3.*
4. *The central bank sets the interbank rate i according to rule (34) and sets the interest on reserves $i^R \leq i$. Its budget constraints (36) and (38) hold.*
5. *The following market-clearing conditions for goods and asset markets hold:*

$$C_0 = Y - f^{-1}\Big(K_{C,0} + K_{B,0} + K_{CB}\Big) \tag{62}$$

$$\phi \cdot C_1 + Q_1 \cdot K_{C,1} = L_C + L_B + K_{CB} \tag{63}$$

$$(1-\phi) \cdot C_2 = (1+r) \cdot [(1+r) \cdot (K_{C,0} + K_B) - (L_C - L_B) + K_{C,1}] \tag{64}$$

$$K_{C,0} + K_B + K_{CB} = K^S(Q_0) \tag{65}$$

$$K_{C,1} = K^S(Q_1) \tag{66}$$

$$D_C(d_0, d_1; \nu) = D \tag{67}$$

$$R + M = OMO \tag{68}$$

$$F = 0. \tag{69}$$

Definition 8 (Sticky-price equilibrium).
The sticky-price equilibrium consists of quantities

$$(C_0, C_1, C_2, K_{C,0}, K_B, K_{CB}, K_{C,1}, I_0, I_1, L_C, L_B, D, M, R, F, OMO, \Pi_B, \Pi_F, T)$$

and prices

$$(P_0, P_1, Q_0, Q_1, d_0, d_1, i^R, i)$$

such that

1. *Firms solve their optimisation problem as in definition 1.*
2. *Consumers solve their optimisation problem as in definition 2.*
3. *Banks solve their optimisation problem as in definition 3.*

4. *The central bank sets the interbank rate i according to Taylor rule (34) and sets the interest on reserves $i^R \leq i$. Its budget constraints (36) and (38) hold.*

5. $P_1 = P_0 = \overline{P}$.

6. $C_0 \leq Y - f^{-1}\Big(K_{C,0} + K_B + K_{CB}\Big)$.

7. *The following market-clearing conditions for goods and asset markets hold:*

$$\phi \cdot C_1 + Q_1 \cdot K_{C,1} = L_C + L_B + K_{CB} \tag{70}$$

$$(1-\phi) \cdot C_2 = (1+r) \cdot [(1+r) \cdot (K_{C,0} + K_B) - (L_C - L_B) + K_{C,1}] \tag{71}$$

$$K_{C,0} + K_B + K_{CB} = K^S(Q_0) \tag{72}$$

$$K_{C,1} = K^S(Q_1) \tag{73}$$

$$D_C(d_0, d_1; \nu) = D \tag{74}$$

$$R + M = OMO \tag{75}$$

$$F = 0. \tag{76}$$

B Capital-Production Sector in Diamond and Dybvig (1983)

The payoffs of capital described in Diamond and Dybvig (1983) can be thought of in our economy as the case in which the capital supply function is perfectly elastic at $Q = 1 + r$.

In their economy capital production is not modelled. The payoffs from holding capital are represented in figure 4. The price of capital in terms of consumption goods is 1, regardless of how much capital is produced. The yield from holding capital for one period is 0 and the yield from holding capital for two periods is $\sqrt{1+r} - 1 > 0$.[23]

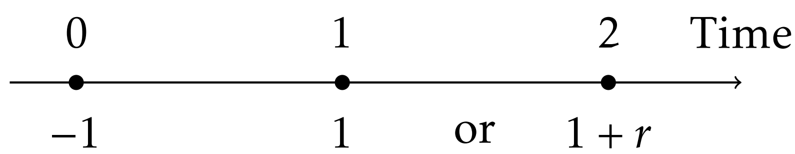


Figure 4: Capital payoffs in Diamond and Dybvig (1983)

The yield curve is what matters in this model for the consumer's saving decision. And the same yield curve is obtained in our framework with $Q = 1+r$. A capital-production function that would deliver such result is

$$K = f^{DD}(I) = \frac{I}{1+r}, \tag{77}$$

where I represents the consumption goods invested to produce capital goods K.

Notice that the marginal product of investment is strictly positive as in f and that $\frac{\partial f^{DD}(0)}{\partial I} = \frac{1}{1+r}$, which complies with the parametric restriction on the production function f. The difference with f is that production function f^{DD} has no decreasing returns $\frac{\partial^2 f^{DD}}{\partial I^2} = 0$.

The purpose of the capital-production sector is to introduce decreasing returns in the model. This is crucial to study monetary policy in a model with capital, because in such model decreasing returns to capital investment are necessary for interest rates to be endogenous. Otherwise, they are pinned down by the constant return to capital. Thus, an interesting framework to study monetary policy, the role of which is to adjust interest rates, requires decreasing returns to capital investment.

[23] Please refer to figure 8 in appendix E for a graphical representation.

C Disintermediated Economy

To understand the role that investment intermediation plays in this model, it is useful to study the sticky-price equilibrium allocation without deposits.[24]

Definition 9 (Sticky-price disintermediated equilibrium).
The sticky-price disintermediated equilibrium is the sticky-price equilibrium given by definition 8 in which banks cannot supply deposits, so that

$$\hat{D} = 0. \tag{78}$$

The disintermediated equilibrium defines an allocation $(\hat{C}_0, \hat{C}_1, \hat{C}_2, \hat{K}_0, \hat{X}, \hat{Q}_0, \hat{\imath})$. Interest on reserves i^R is irrelevant for the allocation , because there is no demand for reserves in the absence of deposits.

Proposition 4.
The allocation in the disintermediated equilibrium is inefficient, as

$$\hat{X} = \phi + (1-\phi) \cdot (1+r)^{1-\gamma} < 1. \tag{79}$$

The equilibrium allocation is inefficient regardless of rationing, because consumers do not insure each other against liquidity risk. The economy is still vulnerable to liquidity-trap episodes, in which rationing takes place. In these cases, there are two sources of inefficiency: rationing and partial liquidity-risk insurance.

[24] In the literature on maturity transformation, this benchmark case is also known as the autarkic equilibrium, wherein agents cannot insure each other against idiosyncratic liquidity risk.

D Proofs

Lemma 1 (**Properties of capital supply**).
The capital supply function has the following properties:

1. $\frac{\partial K^S(Q)}{\partial Q} \in (0, +\infty)$,

2. $K^S(Q) \geq 0 \iff Q > (1+r)^{\frac{\gamma-1}{\gamma}}$.

Proof. Function $K^S(Q)$ is given by equations (3) and (1).

1. Total differentiation gives $\frac{dK}{dQ} = -\frac{f''(I)}{f'(I)}$. Given $f' > 0$ and finite, and $f'' < 0$, we have that $\frac{\partial K^S(Q)}{\partial Q} \in (0, +\infty)$.

2. Given $f'(0) = \lambda < (1+r)^{-\frac{\gamma-1}{\gamma}}$, $f'' < 0$, and twice-continuous differentiability of f, $K^S(Q) \geq 0 \iff Q > (1+r)^{\frac{\gamma-1}{\gamma}}$.

□

Lemma 4 (**Equilibrium deposit contract with slack demand for reserves**).
If $(1+i^R) \cdot \frac{P_0}{P_1} = \frac{1+r}{Q_0}$, *the equilibrium deposit contract features deposit rates*

$$1 + d_0 = \frac{\alpha}{\phi \cdot \alpha + 1 - \phi} \cdot \frac{1+r}{Q_0}, \tag{29}$$

$$(1+d_0) \cdot (1+d_1) = \frac{1}{\phi \cdot \alpha + 1 - \phi} \cdot \frac{(1+r)^2}{Q_0}, \tag{30}$$

with

$$\alpha \in \left[(1+r)^{\frac{\gamma-1}{\gamma}}, Q_1\right]. \tag{31}$$

Proof. The equilibrium deposit contract is such that there are no other contracts that consumers strictly prefer and that firms can supply without making losses.

The zero-profit condition and the fact that $(1+i^R) \cdot \frac{P0}{P_1} = \frac{1+r}{Q_0}$ imply that the equilibrium deposit contract (d_0, d_1) must be such that equations (29) and (30) hold. I proceed by cases, characterising different deposit contracts by different values $\alpha \leq Q_1$.

Case 1: The case of $\alpha < 1$.

These deposit contracts provide negative liquidity-risk insurance. $\alpha < 1$ gives more to late types than to early types. While possible, there would be no demand for such deposit contracts.

Case 2: The case of $\alpha = 1$.

This deposit contract provides no liquidity-risk insurance. It has the same payoffs as the underlying capital. From the consumers perspective, deposits are perfect substitutes with direct capital holdings.

Show that there are contracts on the zero-profit condition that the consumer strictly prefers. Suppose the equilibrium contract supplied has $\alpha = 1$. Then, by budget constraints (9) and (10), $\frac{C_2}{C_1} = 1 + r$. Suppose one bank deviated and offered a deposit contract with $\underline{\alpha} > 1$. This contract would be strictly preferred by the consumer as can be seen from

$$\xi^D - \underline{\xi}^D = \frac{\phi \cdot (1-\phi) \cdot (\underline{\alpha} - 1)}{(\underline{\alpha} \cdot \phi + 1 - \phi) \cdot (1+r)^{\gamma-1}} \cdot \left[(1+r)^{\gamma-1} - 1 \right] > 0, \tag{80}$$

where $\underline{\xi}^D$ is the Kuhn-Tucker multiplier associated with the non-negativity constraint on the deposit contract characterised by $\underline{\alpha}$.

Hence, a deposit contract with $\alpha = 1$ is not the equilibrium deposit contract.

Case 3: The case of $\alpha \in \left[1, (1+r)^{\frac{\gamma-1}{\gamma}} \right)$.

First, I show that in this case consumers do not hold capital directly (i.e., $\xi^K > 0$). Suppose that $\xi^K > 0$, then $\frac{C_2}{C_1} = \frac{1+r}{\alpha}$. Then,

$$\xi^K - \xi^D = \frac{\phi \cdot (1-\phi) \cdot (\underline{\alpha} - 1)}{(\underline{\alpha} \cdot \phi + 1 - \phi) \cdot (1+r)^{\gamma-1}} \cdot \left[(1+r)^{\gamma-1} - \alpha^{\gamma} \right] > 0 \tag{81}$$

confirms that consumers indeed strictly prefer deposits to direct capital holdings if $\alpha \in \left[1, (1+r)^{\frac{\gamma-1}{\gamma}} \right)$.

Second, I show that there always exists a strictly preferred deposit contract on the zero-profit condition. Consider $\underline{\alpha} > \alpha$, with $\underline{\alpha} \in \left[1, (1+r)^{\frac{\gamma-1}{\gamma}} \right)$. This contract would be strictly preferred by the consumer as can be seen from

$$\xi^D - \underline{\xi}^D = \frac{\phi \cdot (1-\phi) \cdot (\underline{\alpha} - \alpha)}{(\underline{\alpha} \cdot \phi + 1 - \phi) \cdot (\alpha \cdot \phi + 1 - \phi) \cdot (1+r)^{\gamma-1}} \cdot \left[(1+r)^{\gamma-1} - \alpha^{\gamma} \right] > 0, \tag{82}$$

where $\underline{\xi}^D$ is the Kuhn-Tucker multiplier associated with the non-negativity constraint on the deposit contract characterised by $\underline{\alpha}$.

Hence, a deposit contract with $\alpha \in \left[1, (1+r)^{\frac{\gamma-1}{\gamma}} \right)$ is not the equilibrium deposit contract.

Case 4: The case of $\alpha \in \left[(1+r)^{\frac{\gamma-1}{\gamma}}, Q_1 \right]$.

First, I show by contradiction that in this case consumers hold capital directly (i.e., $\xi^K = 0$). Suppose that $\xi^K > 0$, then $\frac{C_2}{C_1} = \frac{1+r}{\alpha}$. Then,

$$\xi^K - \xi^D = \frac{\phi \cdot (1-\phi) \cdot (\alpha - 1)}{(\alpha \cdot \phi + 1 - \phi) \cdot (1+r)^{\gamma-1}} \cdot \left[(1+r)^{\gamma-1} - \alpha^{\gamma} \right] \leq 0, \tag{83}$$

which contradicts the hypothesis.

Notice that if $\xi^K = \xi^D = 0$, equations (15) and (17) imply that consumers choose a portfolio of deposits and direct capital holdings such that $\frac{C_2}{C_1} = (1+r)^{\frac{1}{\gamma}}$.

Then, I show that there is no deposit contract on the the zero-profit condition that is strictly preferred. Consider a deposit contract characterised by $\underline{\alpha} \in \left[(1+r)^{\frac{\gamma-1}{\gamma}}, Q_1\right]$ with $\underline{\alpha} \neq \alpha$. The consumer would be indifferent between the two contracts, as can be seen from

$$\xi^D - \underline{\xi}^D = 0 \tag{84}$$

Hence, all deposit contracts characterised by $\alpha \in \left[(1+r)^{\frac{\gamma-1}{\gamma}}, Q_1\right]$ are equilibrium deposit contracts. □

Lemma 5 (**Equilibrium deposit contract with binding demand for reserves**).
If $(1+i^R)\cdot\frac{P_0}{P_1} < \frac{1+r}{Q_0}$, *the equilibrium deposit contract features deposit rates*

$$1 + d_0 = \frac{1}{\phi + (1-\phi)\cdot\frac{1}{Q_1}} \cdot \left[(1-\rho)\cdot\frac{1+r}{Q_0} + \rho\cdot(1+i^R)\cdot\frac{P_0}{P_1}\right] \tag{32}$$

$$1 + d_1 = \frac{1+r}{Q_1} \tag{33}$$

Proof. The equilibrium deposit contract is such that there are no other contracts that consumers strictly prefer and that firms can supply without making losses.

The zero-profit condition and the fact that $(1+i^R)\cdot\frac{P0}{P_1} = \frac{1+r}{Q_0}$ imply that the equilibrium deposit contract (d_0, d_1) must be such that equations

$$1 + d_0 = \frac{\alpha}{\phi\cdot\alpha + 1 - \phi} \cdot \left[(1-\rho)\cdot\frac{1+r}{Q_0} + \rho\cdot(1+i^R)\cdot\frac{P_0}{P_1}\right] \tag{85}$$

and

$$(1+d_0)\cdot(1+d_1) = \frac{1}{\phi\cdot\alpha + 1 - \phi} \cdot \left[(1-\rho)\cdot\frac{1+r}{Q_0} + \rho\cdot(1+i^R)\cdot\frac{P_0}{P_1}\right]\cdot(1+r) \tag{86}$$

hold. I proceed by cases, characterising different deposit contracts by different values $\alpha \leq Q_1$.

Case 1: The case of $\alpha < (1+r)^{\frac{\gamma-1}{\gamma}}$.

Please refer to cases 1 to 3 in the proof of lemma 4 for a demonstration that deposit contracts with $\alpha < (1+r)^{\frac{\gamma-1}{\gamma}}$ are not sustained in equilibrium.

Case 2: The case of $\alpha \in \left[(1+r)^{\frac{\gamma-1}{\gamma}}, Q_1\right]$.

First, I show by contradiction that in this case consumers hold capital directly (i.e., $\xi^K = 0$). Suppose that $\xi^K > 0$, then $\frac{C_2}{C_1} = \frac{1+r}{\alpha}$. Then, define

$$z = 1 - \rho + \frac{\left(1+i^R\right)\cdot\frac{P_0}{P_1}}{\frac{1+r}{Q_0}} < 1 \tag{87}$$

and notice that

$$\xi^K - \xi^D = \frac{\phi \cdot (1-\phi) \cdot (\alpha - 1)}{(\alpha \cdot \phi + 1 - \phi) \cdot (1+r)^{\gamma-1}} \cdot \left\{ \frac{\alpha \cdot \frac{z-\phi}{1-\phi} - 1}{\alpha - 1} \cdot (1+r)^{\gamma-1} - \left[\frac{1-z}{\phi \cdot (\alpha-1)} + 1 \right] \cdot \alpha^{\gamma} \right\} < 0. \quad (88)$$

This contradicts the hypothesis. Hence, consumers hold capital (i.e., $\xi^K = 0$) if $\alpha < (1+r)^{\frac{\gamma-1}{\gamma}}$.

Notice that if $\xi^K = \xi^D = 0$, equations (15) and (17) imply that consumers choose a portfolio of deposits and direct capital holdings such that

$$\frac{C_2}{C_1} = \left[(1+r) \cdot \frac{\alpha - 1 + \frac{1-z}{\phi}}{\alpha \cdot \frac{z-\phi}{1-\phi} - 1} \right]^{\frac{1}{\gamma}}. \quad (89)$$

Second, I show that for $\alpha < Q_1$ there always exists a strictly preferred deposit contract on the zero-profit condition. Consider $\underline{\alpha} > \alpha$, with $\underline{\alpha} \in \left[1, (1+r)^{\frac{\gamma-1}{\gamma}} \right)$. This contract would be strictly preferred by the consumer as can be seen from

$$\xi^D - \underline{\xi}^D = \frac{(1-z) \cdot (\underline{\alpha} - \alpha)}{\phi \cdot \underline{\alpha} + 1 - \phi} > 0. \quad (90)$$

Notice also that deposit contract α is strictly preferred to to any deposit contract with $\underline{\alpha} < \alpha$.

It follows that the equilibrium deposit contract has $\alpha = Q_1$, because no contract with $\underline{\alpha} > \alpha$ can be constructed in this case. □

Proposition 2 (**Feasibility of implementation of first-best allocation**).
There exists a threshold $\overline{\beta}$ such that

1. *If the demand shock $\beta \leq \overline{\beta}$, then $i^R = i$ implements the first-best allocation; and*
2. *If the demand shock $\beta > \overline{\beta}$, then there exists no $i^R(\beta)$ such that the sticky-price equilibrium allocation is first-best. I call this the liquidity trap.*

Proof. Consider the sticky-price equilibrium allocation with $i^R = i$. Full employment, defined as inequality (45) holding with equality, is implemented if and only if $\beta \leq \overline{\beta}$ with

$$\overline{\beta} = \frac{(1+r)^{2\cdot\gamma-1} \cdot \left[K^S\left(\frac{1+r}{1+LB}\right) \right]^{\gamma}}{\left\{ Y - f^{-1}\left[K^S\left(\frac{1+r}{1+LB}\right) \right] \right\}^{\gamma} \cdot (1+LB) \cdot \left[\phi \cdot (1+r)^{\frac{\gamma-1}{\gamma}} + 1 - \phi \right]^{\gamma}} \quad (91)$$

□

Proposition 3 (**Optimal Monetary Policy in Liquidity Trap**).
If the economy is in the liquidity trap as defined in proposition (2), the optimal interest rate on reserves sets $i^R < LB$. It implements a second-best allocation with partial liquidity-risk insurance $X \in (\hat{X}, 1)$.

Proof. The optimality condition of the optimal monetary policy problem in the liquidity trap is given by

$$LHS(X) + \mu(X) = RHS(X), \tag{92}$$

with

$$LHS(X) = \frac{\phi \cdot \beta^{\frac{1}{\gamma}} \cdot X^{\frac{1}{\gamma}} \cdot (1 - X)}{(X - \phi) \cdot Z(X)} \tag{93}$$

$$LHS'(X) = -\frac{[(\gamma - 1) \cdot X + \phi] \cdot (1 - X) \cdot Z(X) + (X - \phi) \cdot \gamma \cdot X \cdot [Z(X) + Z'(X) \cdot (1 - X)]}{(X - \phi) \cdot \gamma \cdot X \cdot [Z(X)]^2 \cdot (\beta \cdot X)^{\frac{1}{\gamma}}}$$

$$RHS(X) = \frac{1}{\gamma} \cdot \left[1 - \frac{1 - \phi}{X - \phi} \cdot \frac{X}{Z(X)}\right] \tag{94}$$

$$RHS'(X) = \frac{X \cdot Z'(X) \cdot (X - \phi) + \phi \cdot Z(X)}{(1 - \phi)^{-1} \cdot \gamma \cdot [Z(X)]^2 \cdot (X - \phi)^2}$$

$$Z(X) = 1 - \phi + \phi \cdot \left(\frac{X - \phi}{1 - \phi}\right)^{\frac{1}{\gamma}} \cdot (1 + r)^{\frac{\gamma - 1}{\gamma}} \tag{95}$$

$$\mu(X) = \nu(X) \cdot \frac{X}{C_0(X)} \tag{96}$$

$\mu(X)$ is a transformation of the Kuhn-Tucker multiplier $\nu(X) \geq 0$ associated with complementary-slackness condition

$$\nu(X) \cdot [C_0(X) - Y - Q_0 \cdot K_0] = 0 \tag{97}$$

and function $C_0(X)$ is defined by implementability constraint (58).

First, I look for $\tilde{X}$ over the feasible domain $[\hat{X}, 1]$ such that

$$LHS(\tilde{X}) = RHS(\tilde{X})$$

$\tilde{X}$ is the optimum if $\mu(\tilde{X}) = 0$ (i.e., if inequality constraint (59) is not binding).
Over the domain, $LHS(X)$ is continuous, $LHS'(X) < 0$, $LHS(\hat{X}) > 0$ and LHS(1)=0. Also, $RHS(X)$ is continuous, $RHS'(X) > 0$, $RHS(\hat{X}) < 0$ and $RHS(1) > 0$. It follows that in the domain $X \in [\hat{X}, 1]$ there exists a unique $\tilde{X}$.

I can restrict the interval to which $\tilde{X}$ belongs and show that inequality constraints (60) are not binding. Define $\underline{X} \equiv \phi + (1 - \phi) \cdot (1 + r)^{\frac{1-\gamma}{1+\gamma}} > \hat{X}$. Notice that $RHS(\underline{X}) = 0$. This implies that $\tilde{X} > \underline{X} > \hat{X}$. Moreover, $LHS(1) < RHS(1)$. Hence, $\tilde{X} < 1$. It follows that $\tilde{X} \in (\underline{X}, 1)$ and inequality constraints (60) are not binding.

Second, I look for X^{opt} such that optimality condition (92) holds. If inequality constraint (59) is not binding, thus $\mu = 0$, then $X^{opt} = \tilde{X}$. But if it is binding, then $X^{opt} \neq \tilde{X}$.

Define $\tilde{\tilde{X}}$ such that inequality constraint (59) holds with equality. Notice that the inequality constraint sets a maximum value for C_0. By the definition of liquidity trap, $\tilde{\tilde{X}} < 1$.Moreover, by definition

$$C_0'(X) = -RHS(X) \cdot \frac{C_0}{X}$$

which implies that over the interval $(\underline{X}, 1)$, to which $\tilde{X}$ belongs, C_0 is strictly decreasing in X. Hence, in the interval $(\underline{X}, 1)$ there is at most one $\tilde{\tilde{X}}$ and $X^{opt} \geq \tilde{\tilde{X}}$.

It follows that there exists a unique $X^{opt} = \max\{\tilde{X}, \tilde{\tilde{X}}\}$ and $X^{opt} \in (\hat{X}, 1)$. $\square$

E Figures

Figure 5: Interest rate on excess reserves, 2012-2017

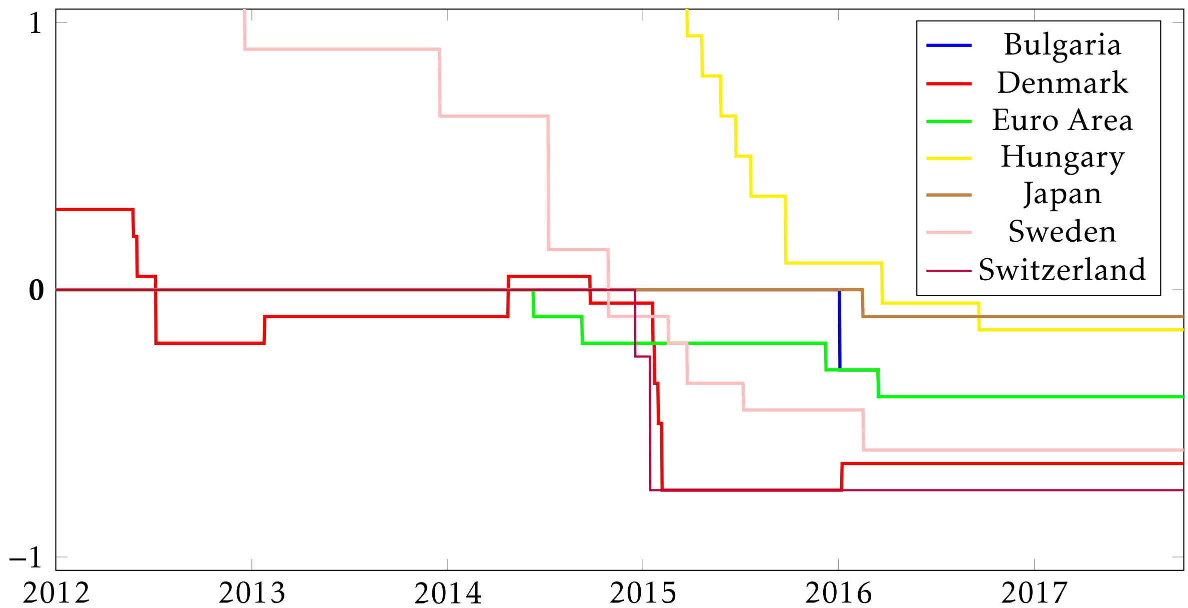


Figure 6: Capital supply

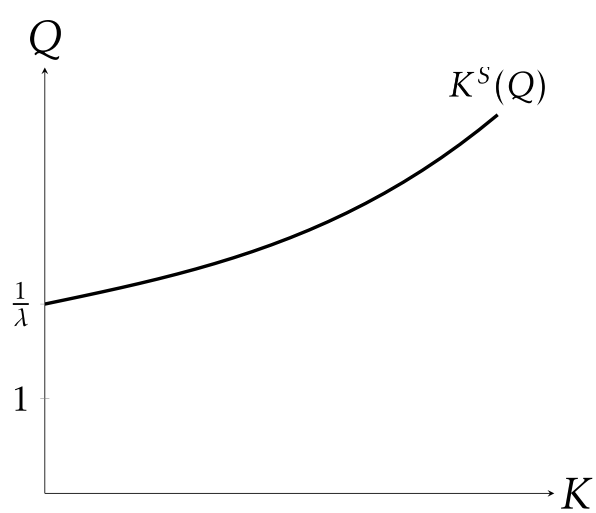


Figure 7: Upward-sloping yield curve

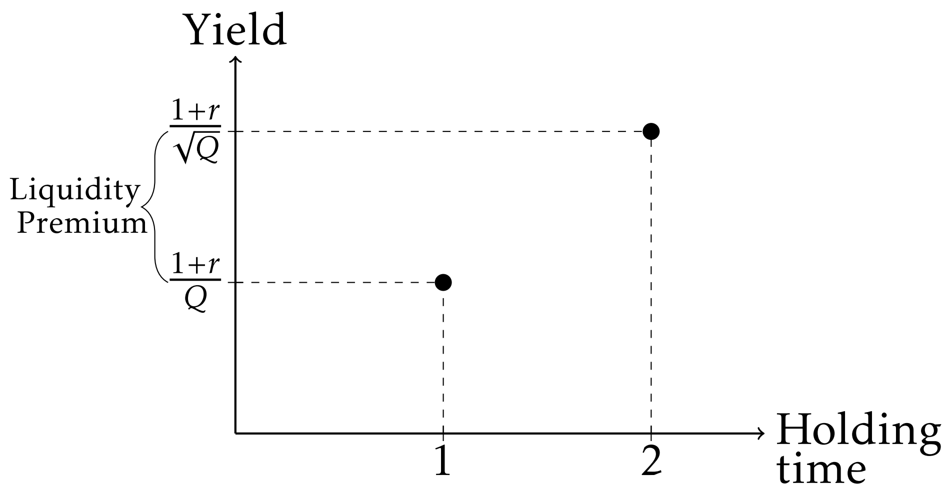


Figure 8: Capital supply in Diamond and Dybvig (1983)