Maturity Transformation and Negative Interest Rate Policies[a]

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Abstract

I provide a theory of maturity transformation with Keynesian elements. By including a lower bound on retail deposit rates, I use the model to study the effects and welfare consequences of negative interest rate policies. From a positive perspective, I find first that deposit rates being bounded at zero do not imply a lower bound on the policy rate. In fact, a cut to the policy rate is expansionary at negative values, too. Second, I find that a negative policy rate distorts the portfolio allocation decision of the banking system. Because a negative policy rate makes deposits more onerous for banks, it forces them to hold a more liquid portfolio, even if illiquid assets yield a higher return. In sum, when the economy needs a negative policy rate to achieve full employment, the central bank faces a policy trade-off between stimulating demand and ensuring that banks do not hold excessively liquid asset portfolios. From a normative perspective, I find that it is optimal for the central bank to solve the trade-off by choosing an interior solution where unemployment and excessive liquidity co-exist. Hence, despite the absence of a lower bound on the policy rate, the model features liquidity traps with unemployment and excessive liquidity.

Keywords: Liquidity, lower bound, maturity transformation, unconventional monetary policies.

JEL Codes: E43, E52, G21.

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1 Introduction

In recent years, an increasing number of central banks have set their policy rate at negative levels. This fact has fundamentally challenged our view of the economic contingency called the liquidity trap.

The modern literature on the liquidity trap, which started with Krugman (1998), was founded on the idea that the presence of currency in the economy made it impossible for the central bank to set a negative policy rate. The latest experience has shown this assumption to be false. A negative policy rate is, at least to some extent, feasible.

Given that a negative policy rate is feasible, it is interesting to understand what effects it has and, ultimately, whether it is desirable. Together with a handful of academic contributions, this paper develops a theory that is well suited to addressing these questions.

At a fundamental level, the answers to these questions depend on whether there are frictions in the economy that are activated or made stronger by the policy rate being negative. In this regard, Heider et al. (2018) and Eggertsson et al. (2019) have persuasively documented that banks face a zero lower bound on the interest rates that they offer on deposits, and that this constraint is more relevant when the policy rate turns negative. The existence of this lower bound on deposit rates is the key premise of this paper and of the rest of the literature on negative interest rate policies.[1]

Because the lower bound on deposit rates is a constraint on banks, models that study the effects of a negative policy rate need to feature a meaningful banking sector. Indeed, the specification of the banking system is a key determinant of the paper's results. This paper is the first to study negative interest rates in a model where banks perform maturity transformation as in the literature started by Diamond and Dybvig (1983). I emphasise the effect of a negative policy rate on the provision of liquidity and the portfolio-allocation decision of the banking sector. The role of the banking sector in the model sets this paper aside from the rest of the literature on negative interest rate policies. Brunnermeier and Koby (2018) and Eggertsson et al. (2019) specify banks as credit intermediaries subject to a leverage constraint, following the seminal paper by Bernanke et al. (1999). These papers emphasise the effect of negative interest rate policies on the bank-lending channel of monetary policy. It is important to stress that these views of banking are not incompatible. In fact, they are complementary. There is no doubt that banks perform both maturity transformation and credit intermediation. Thus, a complete model of banking should incorporate both aspects.

From a positive perspective, I have two results. First, a cut to the policy rate is expansionary at negative levels, even in the presence of a zero lower bound on deposit rates. Monetary policy affects the economy through the classic interest-rate channel, which stimulates investment. Second, a negative policy rate distorts the portfolio allocation of the banking system. A binding lower bound on deposit rates makes the bank's short-term liabilities more onerous. In order to service these obligations, banks invest a larger share of their portfolio in liquid assets. Interestingly, given a negative policy rate, an increase in the liquidity premium reduces the entity of the distortion, because more earnings from

[1] For an interesting exception, see Rognlie (2016). He conceptualises a negative policy rate as a tax on short-term assets that, while expansionary, leads consumers to hold too much paper currency.

maturity transformation make the lower bound on deposit rates slacker.

These positive results imply that, if the economy needs a negative policy rate to achieve full employment, the central bank faces a trade-off between stimulating demand and ensuring that banks do not hold excessively liquid asset portfolios. In contrast, Brunnermeier and Koby (2018) and Eggertsson et al. (2019) find that at negative values of the policy rate the bank-lending channel operates in reverse, as policy-rate cuts reduce the profitability of the banking system. No policy trade-off exists in their framework. A negative interest rate policy simply defeats its purpose by contracting the economy.

From a normative perspective, this paper finds that the expansion of monetary-policy tools to include negative policy rates is unambiguously and strictly beneficial. In fact, following a large adverse demand shock, a benevolent central bank should lower the policy rate into strictly negative territory. Nonetheless, it should not do so to the extent that it completely closes the output gap. Indeed, it should choose an interior solution where less severe distortions in the banking sector are traded off for output. It follows that the model features liquidity traps with unemployment and excessive liquidity, even if the central bank behaves optimally.

The development of a nominal model of monetary policy with a banking sector that endogenously performs maturity transformation à la Diamond and Dybvig (1983) is a contribution of this paper in its own right. While nominal models of maturity transformation have already been written (Allen et al., 2014), to the best of my knowledge this is the first model of this kind that allows an analysis of interest-rate setting, arguably the most important dimension of monetary policy. The key elements for this are endogenous interest rates and endogenous income.

1.1 Related literature

Broadly speaking, this paper is related to two different strands of economic literature. It tackles questions and uses methods that belong in the macroeconomic literature on the liquidity trap. Also, it applies methods and ideas from the finance literature on maturity transformation.

The literature on the liquidity trap deals with economic depressions. The key idea was first formulated by Hicks (1937) and the seminal paper for the modern literature is Krugman (1998). A flurry of papers has followed since. The integration of the zero lower bound on the policy rate within the canonical New Keynesian framework is reached in Eggertsson and Woodford (2003). Typically, the shocks that lead into a depression are modelled as simple time-preference shocks. More structure on these large adverse demand shocks is provided by Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012), who model the adverse demand shock as the consequence of a credit crisis.

A large literature on unconventional monetary policies has followed the liquidity-trap literature both chronologically and conceptually. The question asked is whether monetary-policy instruments, which in this case are not the setting of a positive short-term nominal interest rate, can mitigate the effects of an economic depression. For example, Cúrdia and Woodford (2011) and Gertler and Karadi (2011) study the effects of central bank interventions in credit markets when a liquidity trap is associated with financial distress. Del Negro et al. (2012) study the effects of forward guidance. The theoretical work on negative interest rate policies, which I discussed in the previous section, is a small branch of this literature. Empirical work on the effects of negative interest rate policies has been carried out, too. Heider et al. (2018) are the first to document the existence of a lower bound on deposit rates. They find that the euro-area banks more subject to such constraint, because they fund themselves more with deposits, lose market share in the market for syndicated loans after the introduction of negative interest rate policies. Eggertsson et al. (2019) confirm the existence of a lower bound on deposit rates for Swedish banks. Intriguingly, they report an increase in the interest rates on mortgages supplied by the banks most exposed the negative policy rate. Amzallag et al. (2018) confirm this effect for Italian banks but find it to be small. Basten and Mariathasan (2018) show that the Swiss banks most exposed to the negative policy rate take on more interest-rate risk by performing more maturity transformation. They also lend to riskier firms. Ampudia and Van den Heuvel (2018) find that a cut to the policy rate in negative territory leads to a reduction in the valuation of banks. This is consistent with a binding lower bound on deposit rates. Altavilla et al. (2019) find that the zero lower bound does not apply to deposit rates paid to corporate depositors. It follows that a cut to the policy rate in negative territory is transmitted to firms and therefore stimulates investment.

A theory of banking based on the notion of maturity transformation was formalised by Bryant (1980) and Diamond and Dybvig (1983). The latter paper is the basis for my theory. A subsequent theoretical literature worked on generalising and extending the framework. Especially relevant for this paper are Farhi et al. (2009), who explicitly model competition in the banking sector, and Allen et al. (2014), who add nominal contracts to the framework.

2 Technology and Preferences

This section lays out the assumptions of the model on technology and preferences. I stand by the assumptions made in Diamond and Dybvig (1983), the seminal paper in the literature on maturity transformation, with two main differences.

First, this is not a pure endowment economy. At time 0, consumers have to expend one unit of effort to get each unit of the consumption good E. Effort is costly, as captured by the disutility function $v(E)$ that is twice continuously differentiable and is characterised by $v' > 0$ and $v'' > 0$, the usual Inada conditions.

Second, there is an investment technology, which transforms consumption goods into capital goods, according to function

$$K_S + K_L = f(I). \tag{1}$$

For analytical simplicity, I choose the functional form $f(I) = I^\alpha$ with $\alpha \in (0,1)$. Decreasing returns to capital are necessary to have an endogenous real interest rate in the model. The model encompasses Diamond and Dybvig (1983) in the limit as $\alpha \to 1$.

In the economy, there are two types of capital good. One is a short-term liquid asset which pays back one unit of the consumption good after one period. The other is a long-term illiquid capital good. It pays $R > 1$ units of the consumption good after two periods and, for simplicity, it cannot be liquidated early. Notice that the illiquid asset is more productive. I call this difference in return the liquidity premium.

The economy is inhabited by a unit mass of ex-ante identical consumers. There are three dates and two time periods. At time 0, consumers decide how much effort to expend in acquiring consumption goods E and make a saving decision. On the subsequent dates, consumers consume the proceeds of their time-0 savings.

Consumers' preferences are represented by utility function

$$U(E, C_0, C_1, C_2, \theta) = u(C_0) - v(E) + \beta \cdot [(1-\theta) \cdot u(C_1) + \cdot \theta \cdot u(C_2)], \tag{2}$$

with felicity function u satisfying Inada conditions. β is the discount factor between time 0 and the future. Shocks to β represent demand shocks at time 0. This paper's utility function is different than Diamond and Dybvig (1983)'s in that consumers also enjoy consumption at time 0. This gives a meaningful saving decision that allows us to study monetary policy with the model. For simplicity, I restrict the felicity function to have a constant coefficient of relative risk aversion γ, so that

$$u(C) = \frac{C^{1-\gamma} - 1}{1-\gamma}. \tag{3}$$

A necessary assumption for our analysis is that $\gamma > 1$. Risk aversion needs to be high enough to generate the demand for liquidity-risk insurance that is at the heart of this paper.

Random variable θ represents an idiosyncratic liquidity shock and, as such, it is only known at time 1. It takes on values 0 or 1. At time 0, agents know the probability of the liquidity shock's realisations:

$$Pr(\theta) = \begin{cases} \phi & \text{if } \theta = 0, \\ 1-\phi & \text{if } \theta = 1. \end{cases} \tag{4}$$

A consumer whose realisation for θ is 0 is said to be hit by the liquidity shock. I refer to these consumers throughout the paper as early types and to the other consumers, who have not been hit by the liquidity shock, as late types. There is no uncertainty at time 0 about the share of consumers who will be hit by the liquidity shock. Hence, there is no aggregate risk.

3 Social Planner

The allocation of the social planner is the benchmark for efficiency in the following analysis of the decentralised economy.

The social planner maximises the expected value of aggregate welfare subject to the economy's resource constraints. Informational frictions, which are a key characteristic of the decentralised economy, do not constrain the social planner. It follows that the social planner represents a high standard of efficiency.

Expected aggregate welfare can be written as

$$\frac{C_0^{1-\gamma}-1}{1-\gamma}-v(E)+\beta\cdot\left\{\phi\cdot\frac{[C_1(0)]^{1-\gamma}-1}{1-\gamma}+(1-\phi)\cdot\frac{[C_2(1)]^{1-\gamma}-1}{1-\gamma}\right\}. \tag{5}$$

The economy's resource contraints are given by:

$$C_0+I=E, \tag{6}$$

$$\phi\cdot C_1(0)+(1-\phi)\cdot C_1(1)=K_S, \tag{7}$$

$$\phi\cdot C_2(0)+(1-\phi)\cdot C_2(1)=R\cdot[I^\alpha-K_S]. \tag{8}$$

Consumption must be non-negative at all points in time and for all consumers, as according to

$$C_t(\theta)\geq 0 \quad \forall\, t,\theta. \tag{9}$$

The allocation that solves the social planner's problem is by definition efficient. The system of equations that pins down the efficient allocation of consumption across types and time $\{E,C_0,I,C_1(\theta),C_2(\theta)\}_{\theta\in\{0,1\}}$ is given by:

$$C_0^{-\gamma}=v'(E), \tag{10}$$

$$C_0^{-\gamma}=\frac{\alpha\cdot\beta}{I^{1-\alpha}}\cdot C_1(0)^{-\gamma}, \tag{11}$$

$$\frac{C_2(1)}{C_1(0)}=R^{\frac{1}{\gamma}}, \tag{12}$$

$$C_0+I=Y \tag{13}$$

$$(1-\phi)\cdot C_2(1)=R\cdot[I^\alpha-\phi\cdot C_1(0)], \tag{14}$$

$$C_1(1)=C_2(0)=0. \tag{15}$$

Definition 1.
An allocation is efficient if $\{E,C_0,I,C_1(\theta),C_2(\theta)\}_{\theta\in\{0,1\}}$ *satisfy equations (10), (11), (13), (12), (14) and (15).*

Equation (10) determines the optimal solution to the consumer's consumption-leisure trade-off. A higher marginal utility of consumption makes it optimal for consumers to expend more effort.

Equation (11) is an Euler equation, in that it governs consumers' saving decision. The optimal saving decision at time zero sets the marginal rate of substitution between time 0 and time 1 equal to the corresponding marginal rate of transformation.

Equation (12) is familiar from the optimality conditions in the original Diamond-Dybvig model. It implies that it is optimal to insure liquidity risk in the economy. If consumers do not insure each other, then early types, who are forced to liquidate their asset early, consume too little. In an efficient economy the provision of liquidity-risk insurance implements equation (12).

It is interesting to notice that the efficient allocation also implies an efficient liquidity of capital in the economy. In particular, at the optimum

$$\frac{K_S}{K_L} = \frac{\phi}{1-\phi} \cdot R^{\frac{\gamma-1}{\gamma}}. \tag{16}$$

If no agent is hit by the liquidity shick so that $\phi = 0$, there is no holding of liquid assets. A larger liquidity premium means that more maturity transformation is being performed and thus the bank holds more liquid assets

This quantity is important in this paper's analysis. As we will see, in the decentralised economy it is distorted by the implementation of negative interest rate policies.

4 Decentralised Economy

In this section, I describe the characteristics and illustrate the decisions of the four types of agents who inhabit the decentralised economy: capital-producing firms, consumers, banks and the central bank.

4.1 Capital-Producing Firms

Capital-producing firms purchase consumption goods and transform them into capital goods, which they sell. They are active at time 0 within a perfectly competitive market for capital goods. A representative firm statically maximises its profits Π_F, given by

$$\Pi_f = Q \cdot (K_S + K_L) - I, \tag{17}$$

where Q is the price of capital in terms of consumption goods, I is the quantity of consumption goods purchased by the firm and invested to produce $K_S + K_L$ units of capital goods. They produce capital goods according to technology

$$K_S + K_L = I^{\alpha}, \tag{1}$$

with $\alpha \in (0,1)$.

The firms' first-order condition implies the investment-demand schedule

$$I = \left(\alpha \cdot Q\right)^{\frac{1}{1-\alpha}}, \tag{18}$$

which is upward-sloping in the price of capital. The Diamond-Dybvig model is nested as the limiting case $\alpha \to 1$, where investment demand is perfectly elastic with price of capital $Q = 1$. Decreasing returns to investment, as assumed in this paper, are necessary to make the interest rate endogenous.

4.2 Consumers

Consumers make labour-supply, saving and portfolio-allocation decisions under idiosyncratic liquidity risk. There is a unit measure of consumers, who are identical as of time 0. A consumer's utility function is given by

$$\tilde{U}(E, C_0, C_1, C_2, \theta) = \frac{C_0^{1-\gamma} - 1}{1-\gamma} - v(E) + \beta \cdot \left\{ (1-\theta) \cdot \frac{[C_1(0)]^{1-\gamma} - 1}{1-\gamma} + \theta \cdot \frac{[C_2(1)]^{1-\gamma} - 1}{1-\gamma} \right\}. \quad (2)$$

All consumers dislike supplying labour and enjoy consumption at time 0. Disutilty of labour supply is increasing and at an increasing rate. At time 1, consumers are subject to a privately-observed idiosyncratic liquidity shock θ: with probability $\phi \in (0,1)$ they become early types, with $\theta = 0$, and enjoy consumption only at time 1; with probability $1-\phi$ they become late types, with $\theta = 1$, and enjoy consumption only at time 2. The consumer's expected utility function is therefore

$$U[E, C_0, C_1(0), C_2(1)] = \frac{C_0^{1-\gamma} - 1}{1-\gamma} - v(E) + \beta \cdot \left\{ \phi \cdot \frac{[C_1(0)]^{1-\gamma} - 1}{1-\gamma} + (1-\phi) \cdot \frac{[C_2(1)]^{1-\gamma} - 1}{1-\gamma} \right\}. \quad (19)$$

At time 0, each consumer receives transfers from the government T, and profits from banks Π_b and firms Π_f.

Consumers are subject to a trade-before-consumption constraint. Consumers cannot consume or deposit directly the consumption goods that they produce. They need to sell them and use the income that they get from the sales Y to buy consumption goods or deposit or hold currency. Clearly, consumers cannot sell consumption goods they do not have, so that

$$Y \leq E. \quad (20)$$

This can be thought as a reduced-form representation of differentiated goods with consumers able to produce only one kind. It is a necessary assumption to make market prices, that are potentially sticky, matter for output. Otherwise, if consumers are stuck with an excessively high price, so that there is not enough demand for their good, they could avoid the friction by just producing for themselves.

Consumers use their income to consume and save. Savings are held in a bank deposit D or in currency Cu. Both assets are nominal and the price of consumption goods in terms of the numéraire is P_t. So, the time-0 budget constraint is given by

$$P_0 \cdot C_0 + D + Cu = P_0 \cdot (Y + T + \Pi_b + \Pi_f). \quad (21)$$

The consumer is assumed to be able to hold currency at no cost. This assumption is justified by the empirical observation that banks in economies where negative interest rates prevail in money markets have refrained from imposing negative interest rates on retail depositors. The role of the assumption is to set a lower bound on to the deposit rate that consumers will accept on their deposit accounts. For the results of the paper, it does not matter that the marginal cost for consumers is precisely zero. The key is that banks' marginal cost of holding currency is higher than consumers'. From a theoretical point of view, this can be rationalised with an increasing marginal cost function for currency holdings. The marginal cost of storing each depositor's wealth individually in currency is smaller than the bank's marginal cost of storing all of its short-term assets in currency.

At time 1, the idiosyncratic liquidity shock is realised. Thus, consumer's decisions at time 1 are contingent on her type realisation. The budget constraint at time 1 is given by

$$P_1 \cdot C_1(\theta) = W(\theta) + Cu. \tag{22}$$

To finance consumption $C_1(\theta)$, she can use her currency holdings Cu or withdraw W from her bank deposit. Without loss of generality, I do not allow consumers to hold currency between time 1 and time 2. It can be shown that in equilibrium they would never want to do so.

At time 2, consumers use their remaining deposits to purchase consumption goods $C_2(\theta)$ according to

$$P_2 \cdot C_2(\theta) = (1 + d_1) \cdot \Big[(1 + d_0) \cdot D - W(\theta)\Big]. \tag{23}$$

d_0 and d_1 are the nominal deposit rates, respectively from time 0 to time 1 and from time 1 to time 2, offered by the deposit contract.

Consumers are also constrained by non-negativity constraints on currency holdings, deposits and consumption:

$$Cu \geq 0, \tag{24}$$

$$D \geq 0, \tag{25}$$

$$C_t(\theta) \geq 0 \quad \forall\, t, \theta. \tag{9}$$

Before I present the bank's problem, it is useful to work out consumers' optimal decisions with regard to withdrawing and depositing. The consumer's withdrawing behaviour is simple. Early types withdraw all of their deposits early. Late types not only do not withdraw their deposit balance. If they stored currency at time 0, then at time 1 they deposit it with their bank. This is formalised as

$$W(\theta) = \begin{cases} (1 + d_0) \cdot D & \text{if } \theta = 0, \\ -Cu & \text{if } \theta = 1. \end{cases} \tag{26}$$

Because in equilibrium depositors will not hold any currency, we can take this as confirmation that consumers withdraw according to type. By not giving late types an alternative saving device between time 1 and time 2, I have abstracted from the incentive-compatibility constraint that features prominently in the literature on bank runs. It can be shown that the incentive-compatibility constraint would be slack in the model's equilibrium.

The demand for bank deposits can be derived from the first-order conditions associated with bank deposits and currency holdings. The consumer makes a portfolio allocation decision between these two assets and decides how much to save overall. The first-order condition associated with deposits is given by

$$u'(C_0) = (1 + d_0) \cdot \frac{P_0}{P_1} \cdot \beta \cdot \{\phi \cdot u'[C_1(0)] + (1 - \phi) \cdot (1 + d_1) \cdot u'[C_2(1)]\} + \xi. \tag{27}$$

where ξ is the Kuhn-Tucker multiplier such that $D \cdot \xi = 0$.

The first-order condition associated with currency holdings is given by

$$u'(C_0) = \frac{P_0}{P_1} \cdot \beta \cdot \{\phi \cdot u'[C_1(0)] + (1 - \phi) \cdot (1 + d_1) \cdot \beta \cdot u'[C_2(1)]\} + \hat{\xi}. \tag{28}$$

where $\hat{\xi}$ is the Kuhn-Tucker multiplier such that $Cu \cdot \hat{\xi} = 0$.

It is worth summarising in a lemma the direct implications of the two equations above for the consumer's portfolio-allocation decision.

Lemma 1. *If and only if $d_0 < 0$, then we have that $D = 0$.*

If the deposit rate falls below zero, no consumer would deposit her savings at a bank. The consumer would rather hold her savings in currency and deposit the currency in the following period. This can be thought of as a constraint on the deposit contract that the bank can offer. The question of this paper is how the presence of this constraint affects the transmission of monetary policy and thus optimal monetary policy.

Another implication of equations (27) and (28) is that the consumer's saving decision can be characterised by the following Euler equation

$$u'(C_0) = \max\left\{1 + d_0, 1\right\} \cdot \frac{P_0}{P_1} \cdot \beta \cdot \{\phi \cdot u'[C_1(0)] + (1-\phi) \cdot (1 + d_1) \cdot \frac{P_1}{P_2} \cdot \beta \cdot u'[C_2(1)]\}. \tag{29}$$

4.3 Central Bank

The central bank sets the interest rate on reserves i and supplies the monetary base M, defined as

$$M = Cu + B. \tag{30}$$

The central bank has no control over the split between bank reserves and currency. The two can be freely exchanged one to one.

At time 0, the central bank issues money by carrying out open-market operations. So, the central bank's time-0 budget constraint is given by

$$P_0 \cdot Q \cdot K_g + P_0 \cdot T = M. \tag{31}$$

The central bank rebates seigniourage revenue to consumers as a lump-sum transfer T.

At time 1, when money is redeemed, the central bank uses its asset holdings to honour its obligations, according to

$$(1 + i) \cdot M - i \cdot Cu = P_1 \cdot K_g. \tag{32}$$

In the last period, the central bank plays no role in the economy.

4.4 Bank

The bank maximises the future utility of depositors

$$\phi \cdot \frac{[C_1(0)]^{1-\gamma} - 1}{1-\gamma} + (1-\phi) \cdot \frac{[C_2(1)]^{1-\gamma} - 1}{1-\gamma}. \tag{33}$$

This is a common assumption in the literature on maturity transformation, starting with Diamond and Dybvig (1983), and it can be shown that these banks are isomorphic to perfectly-competitive profit-maximising banks.

At time 0, the bank offers a deposit contract characterised by interest rates (d_0, d_1) and supplies as many deposits as are in demand. It uses the resources that it collects to invest in capital assets K_b and bank reserves B, the former are real and the latter nominal. The budget constraint at time 1 is given by

$$P_0 \cdot Q \cdot K_b + B = D. \tag{34}$$

At time 1, the bank uses its reserves B and liquidates its capital goods L_b to satisfy total early withdrawal requests. From (26) we know that consumers withdraw according to their type. The budget constraint is therefore

$$\phi \cdot (1 + d_0) \cdot D = P_1 \cdot L_b + (1 + i) \cdot B, \tag{35}$$

where i is the nominal interest rate paid on reserves.

Here, we can already notice that there is potential for arbitrage between reserves and short-term holdings of the capital asset. In equilibrium, this implies the arbitrage condition

$$\frac{1}{Q} = (1 + i) \cdot \frac{P_0}{P_1}. \tag{36}$$

If the real interest on reserves were below the real return from buying a capital asset and liquidating it after one period, the bank would borrow infinitely from the central bank and this is plainly incompatible with equilibrium. The same argument can be made for the opposite inequality.

Time-2 deposit withdrawals are met with the bank's remaining assets, according to

$$(1 - \phi) \cdot (1 + d_0) \cdot (1 + d_1) \cdot D = P_2 \cdot R \cdot (K_b - L_b). \tag{37}$$

A banking contract cannot offer a negative nominal deposit rate, as per lemma 1. Or else, consumers would respond by not depositing. What this implies is that banks are restricted in the deposit contracts they can offer. In particular, we can write that banks are subject to constraint

$$d_0 \geq 0. \tag{38}$$

By using the consumers budget constraints, (22) and (23), and the bank's budget constraints, we can write the non-negativity constraint on the deposit rate as a constraint on the consumption allocation that the bank can implement:

$$\frac{C_2(1)}{C_1(0)} \leq \frac{1 - \phi + i}{1 - \phi} \cdot R. \tag{39}$$

I call this the feasibility constraint. Deposit contracts that offer a relatively low payoff to early depositors are not feasible if they require a negative deposit rate d_0, because a negative deposit rate makes consumers not deposit in the first place. Interestingly, the constraint becomes tighter as the policy rate is reduced. This is because the short-term nominal interest rate is a key determinant of the nominal deposit rate. A low policy rate implies that banks would like to set a low deposit rate and, hence, a sufficiently low policy rate makes banks want to set a negative deposit rate.

It is easy to write the bank's intertemporal budget constraint and, by using the consumers budget constraints (22) and (23), to express it in terms of the consumption pattern that the deposit contract implements. It is as follows:

$$\phi \cdot C_1(0) + (1-\phi) \cdot \frac{C_2(1)}{R} = K. \tag{40}$$

In summary, the bank's problem can be written simply as a choice of future consumption pattern $(C_1(0), C_2(1))$ to maximise (33) subject to the feasibilty constraint (39) and the intertemporal budget constraint (40).

The bank's first-order condition imply

$$\frac{C_2(1)}{C_1(0)} = \begin{cases} R^{\frac{1}{\gamma}} & \text{if } i \in [\underline{i}, +\infty) \\ \frac{1-\phi+i}{1-\phi} \cdot R & \text{otherwise.} \end{cases} \tag{41}$$

The bank behaviour changes when the policy rate falls below $\underline{i}$. In fact, this is the level of the policy rate at which the feasibility constraint becomes binding. I can characterise such threshold as

$$\underline{i} = -(1-\phi) \cdot \frac{R^{\frac{\gamma-1}{\gamma}} - 1}{R^{\frac{\gamma-1}{\gamma}}}. \tag{42}$$

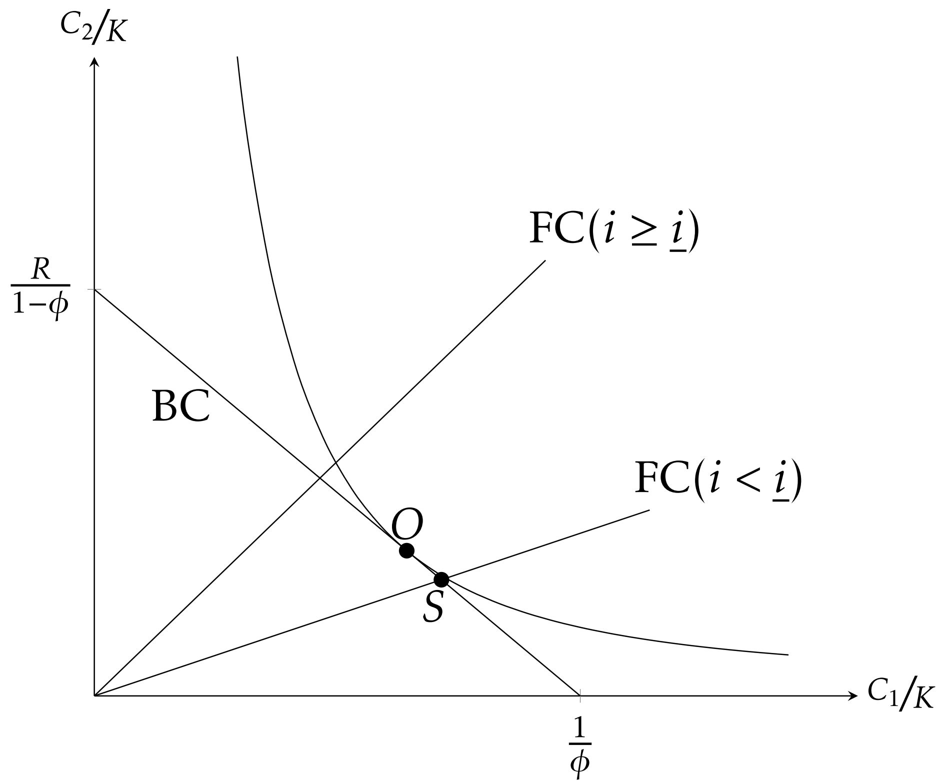
This is not a lower bound on the policy rate. The policy rate may go below this threshold and, as I will show in the next chapter, an interest-rate cut below this threshold remains expansionary. However, this is the threshold below which monetary policy distorts the contracts offered by the economy's financial system. There are two key properties of $\underline{i}$ that are worth noting:

1. $\underline{i} < 0$ and
2. $\underline{i}$ is strictly decreasing in the term premium R.

The former property speaks directly to the debate about negative rates. Slightly negative rates do not distort the decisions of banks, even if the lower bound on deposit rates is zero. The latter property can give us intuition into this result. The larger the term premium in the economy, the lower the policy rate that banks can face without being bound by the feasibility constraint. Since banks can invest in longer-term assets that pay a higher return than short-term assets and therefore can use part of the term premium to remunerate their short-term depositors, they are not constrained by the zero lower bound on deposit rates at slightly negative levels of the policy rate. This result is strictly driven by maturity transformation. Notice that, if the term premium goes to 0 (i.e., $R = 1$) or if liquidity risk goes to zero (i.e., $\gamma = 1$ or $\phi = 1$), then the threshold $\underline{i}$ goes to 0, too. On the second property, it is interesting that a flatter slope of the yield curve worsens the the trade-off of negative interest-rate policies. In this model, the slope of the yield curve is exogenous. Nonetheless, the result hints at interactions between monetary policies aimed at flattening the yield curve and negative interest-rate policies.

I offer a graphical representation of the bank's problem in figure 1. Point O represents the consumption pattern implied by the bank's optimal deposit contract in the absence of

Figure 1: The bank's problem.



the feasibility constraint. In fact, it is the point where the budget constraint is tangent to an indifference curve. The graph shows the feasibility constraint, which becomes tighter as the policy rate is reduced. The key message is that a sufficiently high policy rate implies a non-binding feasibility constraint. If the policy rate is below the threshold $\underline{i}$, then the feasibility constraint is binding and the deposit contract on offer implies consumption pattern S, which is different from O. Specifically, it is different in that early types consume more relative to late types.

At this point, it is important to provide intuition about the distortion and its effects. The consequence of a binding feasibility constraint is that banks liquidate more of their capital. In fact, the average duration of assets is given by

$$\delta = 2 - \frac{R^{\frac{\gamma-1}{\gamma}}}{\phi \cdot R^{\frac{\gamma-1}{\gamma}} + 1 - \phi + \min\{0, i - \underline{i}\}} \cdot \phi. \tag{43}$$

Below the threshold $\underline{i}$, further reductions to the policy rate decrease the average duration of assets in the economy. There is an optimal duration of assets, given by equation (**??**), and going below it is detrimental to welfare, because, while short-term assets provide consumption to consumers hit by the liquidity shock, longer-term assets have a higher return.

5 Equilibrium

I start by collecting the equilibrium conditions derived in the previous section for the model's key variables, $\{I, S, C_0, C_1(\theta), C_2(\theta)\}_\theta$, given prices and monetary policy $\{P_0, P_1, i\}$. The ensuing subsections are dedicated to the behaviour of prices. Therein, I study first the equilibrium with flexible prices. Then, I turn to sticky prices and the implications of this assumption for the equilibrium.

Let us start from the last period. Consolidating all budget constraints, we have that the resource constraint at time 1 and 2 must hold in equilibrium:

$$(1-\phi)\cdot C_2(1) = R\cdot[f(I) - \phi\cdot C_1(0)]. \tag{14}$$

If the short term interest rate falls below a threshold

$$\underline{i} = -(1-\phi)\cdot\frac{R^{\frac{\gamma-1}{\gamma}}-1}{R^{\frac{\gamma-1}{\gamma}}}, \tag{44}$$

then the lower bound on deposit rates is binding and the deposit contracts that banks offer are distorted. The provision of liquidity-risk insurance can be summarised by

$$\frac{C_2(1)}{C_1(0)} = \begin{cases} R^{\frac{1}{\gamma}} & \text{if } i \geq \underline{i}, \\ \frac{1-\phi+i}{1-\phi}\cdot R & i < \underline{i}. \end{cases} \tag{45}$$

Of course, consumers do not consume at the dates when they do not enjoy consumption, so that

$$C_1(1) = C_2(0) = 0. \tag{15}$$

Now, let us consider time 0, when the consumer makes a saving decision and banks make an investment decision. The consumer's saving decision can be described by the Euler equation

$$C_0^{-\gamma} = \max\{1+i, 1+\underline{i}\}\cdot\frac{P_0}{P_1}\cdot J(i)\cdot\beta\cdot C_1^{-\gamma}. \tag{46}$$

If the short-term interest rate decreases, then the consumer saves less ad consumes more, ceteris paribus. However, there is a lower bound on the level of the policy rate that directly affects the consumer's behaviour, since she can switch to currency that pays a zero nominal return. If the short-term rate of interest falls below the threshold $\underline{i}$, then this affects the consumer's saving decision indirectly in two ways. First, because banks ability to provide the optimal quantity of liquidity-risk insurance is curtailed, consumers engage in a precautionary saving behaviour. It is captured by function $J(i)$ defined as

$$J(i) = \begin{cases} 1 & \text{if } i \geq \underline{i}, \\ \frac{R^{\frac{\gamma-1}{\gamma}}}{\phi\cdot R^{\frac{\gamma-1}{\gamma}}+1-\phi}\cdot\frac{\phi\cdot\left[R\cdot\frac{1+i-\phi}{1-\phi}\right]^{\gamma-1}+1-\phi}{\left[R\cdot\frac{1+i-\phi}{1-\phi}\right]^{\gamma-1}} & \text{if } i < \underline{i}. \end{cases} \tag{47}$$

Second, since a ceteris-paribus policy-rate cut implies more investment and thus more future consumption, it makes consumers demand more consumption today, too.

Investment demand is driven by the bank's decision to buy capital, according to

$$(1+i)\cdot\frac{P_1}{P_0}=\frac{\alpha}{I^{1-\alpha}} \tag{36}$$

Banks arbitrage between the interest on reserves and the return on financial markets. In equilibrium, it must be that the two are the same.

Consumption and investment add up to the quantity of goods that are sold in the economy, according to

$$C_0+I=Y \tag{48}$$

and of course not more goods can be sold than the economy is endowed with, i.e.

$$Y\leq E. \tag{49}$$

The difference between the endowment and income are wasted resources. To aid interpretation, in what follows I will refer to it as unemployment. In the continuation of this section, first I analyse the simple case of flexible prices. In this case, sellers have an incentive to change their price until they sell all of their endowment. Then, I move on to a more interesting case: price stickiness.

5.1 Flexible Prices

In the equilibrium with flexible prices, prices are set so that markets clear. This includes

$$Y=E. \tag{50}$$

The logic is that an individual seller has an incentive to undercut any price higher than the price at which she sells all of her merchandise.

In this set up, monetary policy has no role in managing demand, because equilibrium quantities are solely determined by supply. Inflation adjusts until the real rate of interest is consistent with market clearing.

Lemma 2. *Given a monetary policy i, there is a unique short-term real rate of interest, $\frac{1+i}{P_1/P_0}$, consistent with the flexible-price equilibrium.*

This interest rate is useful in the analysis of the sticky-price equilibrium, since it is the real interest rate that is conducive to full employment. Adopting the terminology used in the literature, I call it the natural rate of interest.

Definition 2. *Define the real interest rate that prevails in the flexible-price equilibrium, $\frac{1+i}{P_1/P_0}$, the natural rate of interest. It is indicated by $1+r^n$.*

In the flexible-price equilibrium the only job of the central bank is to keep the short-term nominal interest rate away from the threshold below which it interferes with banks supply of the optimal deposit contract. In other words, the central bank has no reason to set the policy rate i so low that it distorts banks' decisions. Hence, it is optimal for the central bank to refrain from doing so. Optimal monetary policy and the optimal equilibrium allocation can be summarised as follows.

Proposition 1. *With flexible prices, optimal monetary policy prescribes*

$$i \geq \underline{i} \tag{51}$$

and implies an optimal equilibrium allocation $\{C_0, I, C_1(\theta), C_2(\theta)\}_{\theta\in\{0,1\}}$ governed by equations (11), (13), (12), (14) and (15). As according to definition 1, this optimal equilibrium allocation is efficient.

5.2 Sticky Prices

I choose a tractable form of price stickiness:

$$P_1 = P_0. \tag{52}$$

It is a limiting case of staggered pricing à la Calvo (1983). At time 1, the seller cannot change the price she set at time 0. If we interpret the first period to be sufficiently short, this seems to be an approximately good assumption. Notice that I set no restriction to the setting of prices at time 2. Thus, time 2 can be interpreted as the long run, when prices are flexible.

Optimal monetary policy is challenging in this setting because, on the one hand, the nominal rigidity creates a role for monetary policy in managing demand and, on the other hand, low interest rates distort the portfolio allocation of the banking sector.

The threshold on the policy rate below which the economy features excess liquidity is strictly negative. Hence, if positive values of the interest rate ensure full employment of the endowment (i.e., $Y = E$), then the central bank faces no trade-off between employment and liquidity. The trade-off only appears when the natural rate of interest falls into negative territory. Even then, as long as r^n is larger than $\underline{i}$, the central bank can both attain full employment and maintain a healthy banking system. Clearly, in these states of the world the central bank should target the natural rate of interest with its policy rate and thus implement the efficient allocation.

However, there are cases in which setting the interest rate to $\underline{i}$ would still provide insufficient stimulus to obtain full employment. Then, the central bank faces a trade-off. A further cut to the policy rate is expansionary but it distorts bank's provision of liquidity. A sufficiently large discount factor (i.e., a large adverse demand shock) is associated with this state of the world, which I call the liquidity trap.

Lemma 3. *There exists a threshold $\underline{\beta}$ such that, if $\beta > \underline{\beta}$, then $r^n < \underline{i}$. I call this state of the world a liquidity trap.*

Even if the central bank could achieve full employment, we can show that the central bank optimally chooses an interior solution. At the optimum, some of the endowment is wasted for lack of demand and bank portfolios are excessively tilted towards short-term assets.

Proposition 2. *Optimal monetary policy in the sticky-price equilibrium is given by*

$$i = \begin{cases} r^n & \text{if } r^n \leq \underline{i}, \\ \left(r^n, \underline{i}\right) & \text{if } r^n > \underline{i}. \end{cases} \tag{53}$$

The equilibrium allocation is not efficient and it is characterised both by unemployment, with $Y < E$, and by excess liquidity, with $\delta < \delta^*$.

6 Conclusion

In this paper, I developed a model closely based on Diamond and Dybvig (1983), where banks optimally perform maturity transformation. In order to study interest-rate setting by the central bank in this context, I add (1) endogenous real interest rates, (2) endogenous income, (3) nominal contracts, and (4) sticky prices. A theory that allows the study of monetary policy in a context of endogenous maturity transformation is per se a contribution to the literature.

I applied this model to the study of the effects and welfare consequences of a negative policy rate. A negative policy rate has unconventional effects, because banks are subject to a zero lower bound on the interest rate that they offer to depositors, and this constraint may become binding when the policy rate is lowered into negative territory.

The paper has two positive results. First, the effects of a cut to the policy rate on output are weakened in negative territory but remain expansionary. In particular, the interest-rate channel through investment remains active. Second, with a negative policy rate the interest rate that banks pay on their short-term liabilities is higher than it would be if the zero lower bound on deposit rates had not been binding. Banks react to this by shifting their portfolio towards more liquid assets, which can be more easily used to pay these liabilities off. An interesting corollary to the second result is that a higher liquidity premium reduces the distortion induced by a negative policy rate on the bank's portfolio allocation. The larger return that banks make from maturity transformation makes the zero lower bound on the deposit rate slacker.

These two findings imply that, in those cases in which a negative policy rate is needed to stabilise demand, the central bank faces a trade-off. A cut to the policy rate increases output at the expense of the efficiency of bank's portfolio allocation. The optimal solution to such trade-off is given in the paper's normative result. I find that in such case it is optimal for the central bank to choose an interior solution, in which both output is too low and liquidity excessive. In other words, the central bank should set a strictly negative policy rate but not one that is as low as to completely close the output gap.

A discussion has developed over the extent to which central banks should be prudent in lowering their interest into negative territory for fear of inducing distortions (Dell'Ariccia et al., 2017). The normative result of my paper can be read as a formalisation of this argument. Moreover, the theory pins down the relevant dimension along which a negative policy rate distorts economic outcomes, namely the liquidity of assets. This is relatively easy to measure. A direct policy implication of this paper is that an indicator of the liquidity of the banking system's asset portfolio should be monitored, when the policy rate is in negative territory. While a moderate increase in the indicator is to be expected with a negative interest rate policy in place, an excessive increase is undesirable and should weigh against the decision to implement further policy-rate cuts.

Finally, an interesting topic that this paper does not touch upon directly is the presence of interactions of negative interest rate policies with other so-called unconventional monetary policies. An aspect of this paper's findings that speaks to this is that, given a negative policy rate, an increase in the liquidity premium actually helps banks avoid the distortion caused by the lower bound on the deposit rate. This suggests that policies aimed at compressing the liquidity premium may make negative interest rate policies more costly. Future work should study a model where the term premium is determined in equilibrium, rather than being a parameter. This could shed a more direct light on the question.

References

Allen, Franklin, Elena Carletti, and Douglas Gale. 2014. "Money, financial stability and efficiency." *Journal of Economic Theory*, 149(C): 100–127.

Altavilla, Carlo, Lorenzo Burlon, Mariassunta Giannetti, and Sarah Holton. 2019. "Is there a zero lower bound? The real effects of negative policy rates on banks and firms." Unpublished manuscript.

Ampudia, Miguel, and Skander Van den Heuvel. 2018. "Monetary policy and bank equity values in a time of low interest rates." ECB Working Paper Series 2199, European Central Bank.

Amzallag, Adrien, Alessandro Calza, Dimitris Georgarakos, and Jo ao Sousa. 2018. "Monetary Policy Transmission to Mortgages in a Negative Interest Rate Environment." Unpublished manuscript.

Basten, Christoph, and Mike Mariathasan. 2018. "How Banks Respond to Negative Interest Rates: Evidence from the Swiss Exemption Threshold." CESifo Working Paper Series 6901, CESifo Group Munich.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework." In *Handbook of Macroeconomics*. eds. by J. B. Taylor, and M. Woodford, 1 of Handbook of Macroeconomics: Elsevier, , Chap. 21 1341–1393.

Brunnermeier, Markus K., and Yann Koby. 2018. "The Reversal Interest Rate." NBER Working Papers 25406, National Bureau of Economic Research, Inc.

Bryant, John. 1980. "A model of reserves, bank runs, and deposit insurance." *Journal of Banking & Finance*, 4(4): 335–344.

Calvo, Guillermo A. 1983. "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics*, 12(3): 383–398.

Cúrdia, Vasco, and Michael Woodford. 2011. "The Central-Bank Balance Sheet as an Instrument of Monetary Policy." *Journal of Monetary Economics*, 58(1): 54–79.

Del Negro, Marco, Marc Giannoni, and Christina Patterson. 2012. "The forward guidance puzzle." Staff Reports 574, Federal Reserve Bank of New York.

Dell'Ariccia, Giovanni, Vikram Haksar, and Tommaso Mancini-Griffoli. 2017. "Negative Interest Rate Policies – Initial Experiences and Assessments." International Monetary Fund.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

Eggertsson, Gauti B., Ragnar E. Juelsrud, Lawrence H. Summers, and Ella Getz Wold. 2019. "Negative Nominal Interest Rates and the Bank Lending Channel." NBER Working Papers 25416, National Bureau of Economic Research, Inc.

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach." *The Quarterly Journal of Economics*, 127(3): 1469–1513.

Eggertsson, Gauti B., and Michael Woodford. 2003. "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 34(1): 139–235.

Farhi, Emmanuel, Mikhail Golosov, and Aleh Tsyvinski. 2009. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies*, 76(3): 973–992.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.

Guerrieri, Veronica, and Guido Lorenzoni. 2011. "Credit Crises, Precautionary Savings, and the Liquidity Trap."Technical report, National Bureau of Economic Research, Inc.

Heider, Florian, Farzad Saidi, and Glenn Schepens. 2018. "Life Below Zero: Bank Lending Under Negative Policy Rates." *Review of Financial Studies*, Forthcoming.

Hicks, John R. 1937. "Mr. Keynes and the "Classics"; a Suggested Interpretation." *Econometrica*, 5(2): 147–159.

Krugman, Paul R. 1998. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity*, 29(2): 137–206.

Rognlie, Matthew. 2016. "What Lower Bound? Monetary Policy with Negative Interest Rates." Unpublished manuscript.