A Theory of Liquidity and Interest on Reserves[a]

Davide Porcellacchia[b]

October 9, 2018

Working paper

Link to the latest version

Abstract

A well-known result of the Diamond-Dybvig model is that the existence of financial markets curtails provision of liquidity-risk insurance by the banking system. The model's equilibrium allocation has inefficiently low investment in short-term assets. Farhi et al. (2009) propose imposing a reserve requirement on banks in order to increase investment in short-term assets. In a model where consumers are restricted to investing their wealth in bank deposits, they show that a reserve requirement can implement the efficient allocation. First, I show that the fully efficient allocation can equivalently be implemented by payment of positive interest on reserves. Then, I extend the model to allow consumers to invest their wealth directly in illiquid capital assets as well as in bank deposits. Such generalisation breaks down the equivalence between reserve requirement and interest on reserves. A reserve requirement does not lead to more aggregate investment in short-term assets, because it is effectively a tax on bank deposits and consumers respond to it by reducing their holdings of deposits. The paper's main result is that, in the generalised setting, the efficient level of liquidity-risk insurance can only be implemented by a policy of paying strictly positive interest on bank reserves. Interest on reserves provides an incentive for banks to hold more short-term assets and does not lead to disintermediation.

Keywords: Disintermediation, interest on reserves, maturity transformation, reserve requirement.

JEL Codes: E42, E52, E58, G21.

[a]I would like to thank Gianluca Benigno, Wouter Den Haan, Ricardo Reis and Kevin Sheedy for insightful comments on the paper. All errors are my own doing.

[b]European Central Bank. Email: davide.porcellacchia@ecb.europa.eu

1 Introduction

Consumers hold assets for the purpose of smoothing their consumption over time and over states of the world. An asset that is hard to sell or that goes for a low price if sold before its maturity date is an illiquid asset. This is a poor asset to hold for consumption-smoothing purposes.[1]

Assets are illiquid for a variety of reasons. For example, it takes time to build a plant and, until it is completed, the plant has no value except the resale value of the individual parts. Because of asset illiquidity, financial intermediaries play an important role in the economy. They buy illiquid assets and fund such purchases by selling more liquid assets, in the form of bank deposits, to consumers. This business plan is viable because banks can predict deposit withdrawals. A large literature formalises this notion of banks providing liquidity. The seminal paper is Diamond and Dybvig (1983).

There is a limitation to the profitable provision of liquid assets by the banking system. If we accept that assets are illiquid so that agents end up selling their assets at a low price. Then, a direct consequence is that there are great buying opportunities in the economy. In his critique of the Diamond-Dybvig model, Jacklin (1987) points out that in these circumstances no consumer would keep her wealth in a bank deposit. They would all withdraw their deposits in order to buy the cheap assets on sale. It turns out that, if the market for used capital works frictionlessly, then private provision of liquid assets is impossible.

A strand of the literature has found reasons why the Jacklin critique may not hold fully, so that private provision of liquidity is possible. Diamond (1997) adds limited participation in financial markets, Antinolfi and Prasad (2008) assume that consumers who interact in financial markets are subject to borrowing constraints and Hasman et al. (2014) assume transaction costs in the market for used capital. However, the existence of the frictions that these papers advocate only implies that private liquidity is possible to a degree. Even if this is the case, the Jacklin critique still implies that private provision of liquidity alone is insufficient for full efficiency in the economy. Thus, it is important to study whether policy can improve the economy's liquidity outcomes.

Farhi et al. (2009) take the Jacklin critique seriously and study the effectiveness of monetary policy in improving the equilibrium allocation in the Diamond-Dybvig model. They find that a reserve requirement is effective at making assets more liquid in the economy. As banks hold more short-term assets, the deposits held by consumers become more liquid. In fact, a reserve requirement can implement the first-best efficient allocation. Panetti (2017) includes systemic liquidity risk in a similar model and also concludes that imposing a reserve requirement is optimal. A key assumption of these papers is that consumers do not have a choice of how much of their wealth to hold directly in illiquid assets and how many in bank deposits.

In this paper, first I show that in the same setting as in Farhi et al. (2009) the payment of interest on reserves can equivalently implement the first-best efficient allocation. I stress the effect of interest on reserves on incentives: of course, it increases the returns

[1] For a discussion of the several definitions of liquidity that recur in different literatures, see von Thadden (1999).

from holding short-term assets and therefore the incentive to hold them.

Next, I generalise the model. I allow consumers to invest their wealth directly in illiquid assets at time 0, so that the degree of financial intermediation is endogenously determined. I find that this matters for the effects of monetary-policy instruments. In fact, this generalisation breaks the equivalence between reserve requirement and payment of interest on reserves. In the new setting, the central bank can only implement the first-best allocation by paying a positive interest on reserves. In contrast, a reserve requirement is ineffective.

A reserve requirement is effectively a tax on banks. It is a regulation that mandates banks to hold a portfolio of assets that is suboptimal from the banks' viewpoint and thus reduces the return on banks' asset portfolios. The tax is passed on to depositors in the form of lower returns on deposits. Thus, if consumers can also invest their wealth directly in financial markets, they will respond to the reserve requirement by reducing their investment in bank deposits and holding more illiquid assets instead. I find that such financial disintermediation offsets the improvements in liquidity due to the introduction of a reserve requirement. In other words, the effects of a reserve requirement are that (1) banks hold more short-term assets as a share of their asset portfolio, which implies more liquid deposits, but (2) consumers decide to hold fewer deposits. The net effect is that the average liquidity of consumers' assets does not change.

The key result of this paper is that a central bank should pay a strictly positive rate of interest on bank reserves. Interest on reserves provides an incentive for banks to hold more short-term assets and therefore promotes the creation of more liquid deposits. At the same time, unlike liquidity regulation, it does not represent a tax on the banking system. Therefore, it does not lead to financial disintermediation by consumers. It follows that with interest on reserves the central bank can implement the first-best efficient allocation in the economy.

Especially since the Federal Reserve acquired the authority to pay interest on reserves on 1 October 2008, a large literature has focused on the costs and benefits of such policy. An important result, first developed in Sargent and Wallace (1985), is that payment of the market rate of interest on bank reserves decouples the quantity of money from the determination of inflation. Several papers have built on this to argue that paying interest on reserves frees the central bank to use the size of its balance sheet as an additional policy instrument. Thus, it improves the institution's ability to stabilise the economy (Cúrdia and Woodford, 2011; Ennis, 2014; Reis, 2016). This paper is more closely related to another set of papers in this literature, which points out that payment of interest on reserves eliminates the implicit taxation on monetary instruments. Taxing reserves makes the banking system reduce their holdings of reserves to inefficiently low levels. If the tax is eliminated, banks choose to be satiated in reserves. The Friedman rule prescribes such policy as socially optimal (Ennis and Weinberg, 2007; Cochrane, 2014; Canzoneri et al., 2017). In this paper, the inefficiently low level of short-term assets held by financial institutions is structural, rather than the result of implicit distortionary taxation, and payment of positive interest on reserves is better understood as a subsidy on short-term assets. While the literature that looks back to the Friedman rule concludes that the interest on reserves should be set equal to to the market short-term rate of interest, this paper's conclusion is that the government should push up the market short-term interest rate in order to incentivise banks to hold more short-term assets.

1.1 Related Literature

The Diamond-Dybvig model is mostly used to study the fragility of the banking system. A large literature studies whether and how policy can improve the financial system's resilience (Allen and Gale, 2004; Allen et al., 2014). I abstract from bank runs altogether. The model's bank-run equilibrium can be eliminated by assuming that deposits carry a guarantee by the government or that banks have the right to suspend convertibility. Building on the literature on global games started by Carlsson and van Damme (1993), Rochet and Vives (2004) and Goldstein and Pauzner (2005) obtain a unique equilibrium in which bank runs take place with positive probability. This is an important step forward for the study of the effects of policies in the context of the Diamond-Dybvig model. Integrating global games in this paper's analysis of the effects of interest on reserves would allow a discussion of the policy's impact on the likelihood of crises. I leave this to future work.

2 Technology and Preferences

In this section, I lay out the assumptions on technology and preferences of a standard model of financial intermediation, similar to the model in Diamond and Dybvig (1983) and in Allen and Gale (2004).

The economy is inhabited by a unit mass of ex-ante identical consumers, each of which is indicated by j. Consumers live for three periods. In period 0, each consumer receives an endowment E and decides how to invest it. In period 1 and 2, she consumes.

There is one good in the economy, which can be consumed or invested. There are two investment technologies. The short-term investment technology gives one unit of the good after one period for each unit of goods invested. The long-term investment technology gives $R > 1$ goods after two periods for each unit of goods invested.

Consumers are endowed with preferences represented by utility function

$$U(C_{j,1}, C_{j,2}, \theta_j) = (1-\theta_j)\cdot u(C_{j,1}) + \beta \cdot \theta_j \cdot u(C_{j,2}), \tag{1}$$

where the felicity function u complies with Inada conditions. The discount factor is restricted to $\beta > R^{-1}$. An additional necessary assumption is that the coefficient of relative risk aversion is everywhere weakly greater than 1:

$$\frac{-C \cdot u''(C)}{u'(C)} = \gamma(C) \geq 1 \quad \forall C > 0. \tag{2}$$

Risk aversion needs to be sufficiently high to generate a demand for liquidity-risk insurance. Random variable θ_j represents a liquidity shock and, as such, it is only known at time 1. For simplicity, it only takes on values 0 or 1. At time 0, consumers know the objective probability of the liquidity shock's realisations:

$$Pr(\theta_j) = \begin{cases} \phi & \text{if } \theta_j = 0, \\ 1-\phi & \text{if } \theta_j = 1. \end{cases} \tag{3}$$

Consumers with $\theta_j = 0$ are hit by the liquidity shock. Throughout the paper, I refer to them as early types and to other consumers as late types. The liquidity shocks $\{\theta_j\}_{j\in[0,1]}$ are idiosyncratic. The share of consumers who will be hit by the liquidity shock is known at time 0. So, there is no aggregate risk.

3 Social Planner

The social planner observes the realisations of individual liquidity shocks. Hence, in her maximisation of welfare, the social planner is exclusively constrained by technology; not by incentive-compatibility considerations. It is useful to study the social planner's problem to derive a benchmark against which we can evaluate the efficiency of the equilibrium allocation in the decentralised economy. The social planner's allocation corresponds to the first-best efficient allocation.

The social planner faces a choice of non-negative type-contingent consumption paths $\{C_1(\theta_j), C_2(\theta_j)\}_{\theta_j\in[0,1]}$, investment in the short-term technology K_S and investment in the long-term technology K_L. She maximises aggregate welfare, given by

$$\int_0^1 \mathbb{E}_{\theta_j}\Big\{U\Big[C_{j,1}(\theta_j), C_{j,2}(\theta_j), \theta_j\Big]\Big\}\,\mathrm{d}j = \phi \cdot u[C_1(0)] + (1-\phi)\cdot \beta \cdot u[C_2(1)]. \tag{4}$$

The maximisation is subject to resource constraints. In period 0, the social planner uses the aggregate endowment to invest in the short-term technology and in the long-term technology, according to

$$K_S + K_L = E. \tag{5}$$

In period 1, the output of short-term investments finance early consumption, as given by

$$\phi \cdot C_1(0) + (1-\phi)\cdot C_1(1) = K_S. \tag{6}$$

In the last time period, the social planner uses the proceeds of her long-term investments to finance time-2 consumption, according to

$$\phi \cdot C_2(0) + (1-\phi)\cdot C_2(1) = R \cdot K_L. \tag{7}$$

The social planner's allocation $\{C_1(\theta_j), C_2(\theta_j)\}_{\theta_j\in[0,1]}$ satisfies the following equations:

$$\phi \cdot C_1(0) + (1-\phi)\cdot \frac{C_2(1)}{R} = E, \tag{8}$$

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = \beta \cdot R, \tag{9}$$

$$C_1(1) = C_2(0) = 0. \tag{10}$$

These equations define the first-best allocation of the model.

Definition 1 (First-best efficient allocation)**.** *An allocation is first-best efficient if it satisfies equations (8), (9) and (10).*

Notice that definition 1 represents a high standard of efficiency. Incentive compatibility, an important restriction to risk-sharing in the decentralised economy, does not constrain the social planner.

4 Diamond-Dybvig Model with Hidden Trades

The model described in this section is known as the hidden-trade Diamond-Dybvig model. Its key feature is that consumers can trade one-period bonds at time 1 in an anonymous market. Since this bond market is anonymous, deposit contracts cannot restrict consumers' ability to trade in it; hence the name hidden trades. This model is used as a benchmark to study the possibility of coexistence of maturity-transforming banks with financial markets. The key result of the literature is that the banking system's provision of liquidity-risk insurance is curtailed by the existence of financial markets and therefore the equilibrium allocation of an economy where maturity-transforming banks coexist with financial markets is inefficient.

In the first part of this section, I describe the hidden-trade Diamond-Dybvig model without government intervention. I show that the model's competitive equilibrium allocation is inefficient. There is insufficient investment in short-term assets, which implies insufficient liquidity-risk insurance.

In the second part, I study government policies that can improve on the model's laissez-faire equilibrium allocation. I show that a reserve requirement on the banking system can implement first-best efficiency. This is the central result of Farhi et al. (2009). Moreover, I show that first-best efficiency can equivalently be implemented with the payment of positive interest on reserves.

4.1 No Government Intervention

Two unit masses of agents inhabit the economy: a unit mass of consumers indicated by j and a unit mass of banks indicated by k.

The only source of risk in the economy is idiosyncratic liquidity risk. However, though idiosyncratic, liquidity risk cannot be insured away with contingent bonds because of a friction: private observability of liquidity shocks. Consumer j's type, θ_j, is her own private information. Thus, other agents must rely on what consumer j reveals about the realisation of θ_j. Contingent bonds fail because, regardless of the true realisation, each consumer always has an incentive to claim she was hit by the liquidity shock in order to pocket the bond's payout. Banks emerge as a mechanism to insure liquidity risk.

At time 0, consumers choose how much to deposit in each bank k, $\{D_{j,k}\}_{k\in[0,1]}$. Obviously, the consumer will choose to deposit all her endowment in the bank that offers the best deposit rates $(d_{k,0}, d_{k,1})$. Also, at time 1 the consumer decides how much to withdraw from each deposit $\{W_{j,k}(\theta_i)\}_{k\in[0,1]}$ and how much to lend on the bond market $S_j(\theta_j)$ at price Q. These choices determine a type-contingent path for consumption $\{C_{j,1}(\theta_j), C_{j,2}(\theta_j)\}_{\theta_j\in\{0,1\}}$. The consumer maximises

$$\mathbb{E}[U(C_{j,1}, C_{j,2}, \theta_j)] = \phi \cdot u[C_{j,1}(0)] + (1-\phi) \cdot \beta \cdot u[C_{j,2}(1)], \tag{11}$$

subject to constraints

$$\int_0^1 D_{j,k} \, \mathrm{d}k = E, \tag{12}$$

$$C_{j,1}(\theta_j) + Q \cdot S_j(\theta_j) = \int_0^1 W_{j,k}(\theta_j)\,\mathrm{d}k, \tag{13}$$

$$C_{j,2}(\theta_j) = \int_0^1 (1 + d_{k,1}) \cdot [(1 + d_{k,0}) \cdot D_{j,k} - W_{j,k}(\theta_j)]\,\mathrm{d}k + S_j(\theta_j), \tag{14}$$

$$C_{j,t}(\theta_j) \geq 0 \quad \forall t, \theta_j. \tag{15}$$

Banks are profit-maximising firms that compete to attract deposits by offering a deposit contract, which specifies deposit rates (d_0, d_1). Banks know the demand for deposits and can anticipate consumers' withdrawing behaviour. So, before I lay out the banks' maximisation problem, I shall work out from the consumer's maximisation problem the demand for deposits and the consumers' withdrawing behaviour.

Regarding the consumers' withdrawing behaviour, early-type consumers withdraw everything early as long as they are not better off holding on to their deposits and borrowing in the bond market to finance their time-1 consumption. On the other hand, late types withdraw everything at time 2 as long as they are not better off withdrawing at time 1 and buying bonds. This can be formalised as

$$W_{j,k}(\theta_j) = \begin{cases} (1 + d_{k,0}) \cdot D_{j,k} & \text{if } 1 + d_{k,1} < Q^{-1}, \\ [0, (1 + d_{k,0}) \cdot D_{j,k}] & \text{if } 1 + d_{k,1} = Q^{-1}, \\ 0 & \text{if } 1 + d_{k,1} > Q^{-1}. \end{cases} \tag{16}$$

Notice that there can only be a separating equilibrium, in which consumers of different types withdraw in different time periods, in the indifferent case with $1 + d_{k,1} = Q^{-1}$.

Consumers deposit their endowment with the bank that offers the best deposit contract, $(d_{k,0}, d_{k,1})$. To formalise consumers' demand for deposits, I need to derive the consumers' valuation function for deposit contracts from the consumer's maximisation problem. This is given by

$$\begin{aligned} V_j(d_{k,0}, d_{k,1}) \equiv & \phi \cdot u'[C_{j,1}(0)] \cdot (1 + d_{k,0}) \cdot \max\{1, (1 + d_{k,1}) \cdot Q\} + \\ & + (1 - \phi) \cdot \beta \cdot u'[C_{j,2}(1)] \cdot (1 + d_{k,0}) \cdot \max\{1 + d_{k,1}, Q^{-1}\}. \end{aligned} \tag{17}$$

Using this equation, it is easy to write the consumer's demand for deposits as

$$D_{j,k}(d_{k,0}, d_{k,1}) = \begin{cases} E & \text{if } V_j(d_{k,0}, d_{k,1}) > \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\}, \\ [0, E] & \text{if } V_j(d_{k,0}, d_{k,1}) = \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\}, \\ 0 & \text{if } V_j(d_{k,0}, d_{k,1}) < \max_{z \neq k}\{V_j(d_{z,0}, d_{z,1})\}. \end{cases} \tag{18}$$

This is a Bertrand demand correspondence, where demand is fully absorbed by the firm that offers the best conditions to consumers.

Taking as given the price of bonds Q and the deposit contracts offered by other banks $\{d_{z,0}, d_{z,1}\}_{z \neq k}$, bank k offers a deposit contract $(d_{k,0}, d_{k,1})$, which implies a quantity of deposits and of withdrawals from each consumer $\{D_{j,k}, W_{j,k}(\theta_j)\}_{j \in [0,1]}$, and makes a portfolio decision $(K_{k,S}, K_{k,L})$ in order to maximise profits Π_k.[2] The constraints faced by banks are:

$$K_{k,S} + K_{k,L} + \Pi_k = \int_0^1 D_{j,k}\,\mathrm{d}j\,, \tag{19}$$

$$\int_0^1 W_{j,k}(\theta_j)\,\mathrm{d}j = K_{k,S}, \tag{20}$$

$$(1+d_{k,1})\cdot\left[(1+d_{k,0})\cdot\int_0^1 D_{j,k}\,\mathrm{d}j - \int_0^1 W_{j,k}(\theta_j)\,\mathrm{d}j\right] = R\cdot K_{k,L}, \tag{21}$$

$$(K_{k,S}, K_{k,L}) \geq 0. \tag{22}$$

In equilibrium, consumers maximise their expected utility subject to budget constraints, banks offer deposit contracts to maximise their profits and markets clear. I formalise the equilibrium in the Diamond-Dybvig economy with hidden trade as follows.

Definition 2. *The equilibrium consists of a vector of quantities and a vector of prices, respectively* $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j), K_{k,S}, K_{k,L}\right\}_{(j,k,\theta_j)}$ *and* $\left(Q, \{d_{k,0}, d_{k,1}\}_{k\in[0,1]}\right)$*, such that:*

1. *Consumer j chooses quantities* $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j)\right\}_{(k,\theta_j)}$ *to maximise (11) subject to constraints (12), (13), (14) and (15).*

2. *Bank k chooses quantities* $\left(K_{k,S}, K_{k,L}, \{D_{j,k}, W_{j,k}(\theta_j)\}_{(j,\theta_j)}\right)$ *and a deposit contract* $(d_{k,0}, d_{k,1})$ *to maximise profits* Π_k *subject to constraints (16), (18), (19), (20), (21) and (22).*

3. *The bond market clears:* $\int_{j=0}^1 S_j(\theta_j)\,\mathrm{d}j = 0$.

Key to the provision of liquidity-risk insurance by the banking system is the equilibrium price of bonds in the hidden market, Q. A high rate of return on bonds increases the incentive of those consumers who are not hit by the liquidity shock to withdraw early. This makes it harder for banks to separate early-type consumers from late-type ones and therefore reduces the scope for providing liquidity-risk insurance.

Following a large literature (Jacklin, 1987; Allen and Gale, 2004; Farhi et al., 2009), I find that without government intervention the equilibrium return on bonds is equal to the marginal rate of transformation of the economy between time 1 and time 2: R.

Lemma 1. *In the equilibrium of the Diamond-Dybvig economy with hidden trades, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1}{R} \tag{23}$$

and the prevailing deposit contract is given by

$$(d_0, d_1) = (0, R-1). \tag{24}$$

[2] In principle, profits are distributed to consumers, who own the banks. I do not specify this because bank profits turn out to be 0 in equilibrium.

Proof. Please refer to appendix A. □

The logic is that, if the return in the bond market is greater than the return that banks can make on their assets between time 1 and time 2, then the best deposit contract from the consumer's viewpoint pays off at time 1, regardless of the consumer's type realisation, and thus allows the consumer to exploit the high return in the bond market. At price $Q < \frac{1}{R}$ all consumers want to lend in the bonds market and none wants to borrow. This is impossible in equilibrium.

Similarly, if the interest rate on the bond market is lower than R. The best deposit contract will be such that all consumers hold off withdrawing until time 2. Those consumers who want to consume at time 1 should just take advantage of the low interest rate by borrowing in the bond market. However, more borrowing than lending in the bond market is of course not possible in equilibrium.

Strikingly, it turns out that without government intervention the equilibrium price of bonds is so low that banks are unable to provide any liquidity-risk insurance at all. The allocation in equilibrium corresponds to what the literature on maturity transformation calls autarky: the allocation that would be achieved if consumers did not trade at all.

Proposition 1. *The equilibrium allocation of the Diamond-Dybvig model with hidden trades satisfies equations (8), (10) and*

$$\frac{C_2(1)}{C_1(0)} = R. \tag{25}$$

It is not first best.

Proof. Please refer to appendix A. □

Consumers who are hit by the liquidity shock end up consuming too little relative to late-type consumers. Ex-ante, all consumers would have gained from sharing this risk. The provision of liquidity-risk insurance by banks, which implies investing in short-term assets and paying an above-market return to those depositors who withdraw early, is not profitable because it gives an incentive to late-type consumers not to hold on to their deposits at time 1 but to withdraw early too.

4.2 Interest on Reserves and Reserve Requirement

In this section, I study the effects of government interventions on the model's equilibrium allocation. In particular, I analyse whether appropriate monetary policy can implement the first-best allocation.

I introduce bank reserves that pay interest and a reserve requirement. The former means that the government issues reserves, B, a short-term asset that pays an interest rate $i \geq 0$. The latter policy measure is a requirement imposed by the government on banks to hold a minimum share of their asset portfolios in reserves.

The problem that the government is trying to solve is that banks do not have an incentive to invest a sufficient share of their portfolio in short-term assets and thus to provide the socially optimal amount of liquidity-risk insurance. In equilibrium, too little investment is directed to short-term assets. A reserve requirement compels banks to hold more short-term assets and interest on reserves gives banks incentive to hold more short-term assets.

Importantly, neither of the two proposed policy interventions require an informational advantage for the government in order to be implemented. In particular, the government does not need knowledge of the distribution of types.

The government supplies reserves B and promises to pay on them interest rate i. I assume that it invests in short-term capital, $K_{G,S}$, and uses lumpsum taxes, T, on consumers to balance its budget constraints:

$$K_{g,S} = B_g + T, \tag{26}$$

$$(1+i) \cdot B_g = K_{g,S}. \tag{27}$$

I formalise a reserve requirement adding a constraint to the bank's profit maximisation problem:

$$B_k \geq \rho \cdot \int_0^1 D_{j,k} \, \mathrm{d}j. \tag{28}$$

Since $i \geq 0$, banks weakly prefer holding bank reserves to short-term capital. Thus, the results would be exactly identical if I allowed banks to satisfy the requirement with other short-term assets. Banks would still choose to hold reserves.

The equilibrium with government intervention is different from the equilibrium defined in the previous subsection, because (1) there is a government that supplies reserves by buying other short-term assets, (2) consumers are subject to lumpsum taxation, (3) banks hold reserves and are subject to a reserve requirement. A formalisation of the equilibrium is given by

Definition 3. *Given policy instruments $i \geq 0$ and $\rho \in [0,1]$, the equilibrium consists of a vector of quantities $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j), B_k, K_{k,S}, K_{k,L}, K_{g,S}, T, B_g\right\}_{(j,k,\theta_j)}$ and a vector of prices $\left(Q, \{d_{k,0}, d_{k,1}\}_{k\in[0,1]}\right)$ such that:*

1. *Consumer j chooses quantities $\left\{D_{j,k}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j)\right\}_{(k,\theta_j)}$ to maximise (11) subject to constraints*

$$\int_{k=0}^1 D_{j,k} \, \mathrm{d}k = E - T, \tag{29}$$

   (13), (14) and (15).

2. *Bank k chooses quantities $\left(B_k, K_{k,S}, K_{k,L}, \{D_{j,k}, W_{j,k}(\theta_j)\}_{(j,\theta_j)}\right)$ and a deposit contract $(d_{k,0}, d_{k,1})$ to maximise profits Π_k subject to constraints (16), (18), (28),*

$$B_k + K_{k,S} + K_{k,L} + \Pi_k = \int_{j=0}^1 D_{j,k} \, \mathrm{d}j, \tag{30}$$

$$\int_{j=0}^1 W_{j,k}(\theta_j) \, \mathrm{d}j = K_{k,S} + (1+i) \cdot B_k, \tag{31}$$

   (21) and

$$(B_k, K_{k,S}, K_{k,L}) \geq 0. \tag{32}$$

3. $(K_{g,S}, T, B_g)$ *are such that the government's budget constraints (26) and (27) hold.*

4. *The bond market clears:*
$$\int_{j=0}^{1} S_j(\theta_j)\,\mathrm{d}j = 0. \tag{33}$$

5. *The market for reserves clears:* $\int_{k=0}^{1} B_k\,\mathrm{d}k = B_g$.

The impact of the policy instruments on the interest rate prevailing in the bonds market at time 1 is key. Remember that a high interest rate in the bonds market makes banks ineffective at providing liquidity-risk insurance, since it gives an incentive to all depositors, regardless of their type, to withdraw their deposits early. Interest on reserves and the reserve requirement succeed in reducing returns in the bond market by increasing investment in the short-term asset and therefore the quantity of resources available at time 1.

Lemma 2. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*
$$Q = \frac{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}{R}. \tag{34}$$
and the prevailing deposit contract is given by:
$$1 + d_0 = \max\left\{1, \frac{\rho}{\phi}\right\}\cdot(1+i), \tag{35}$$
$$1 + d_1 = \frac{R}{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{36}$$

Proof. Please refer to appendix A. □

If $i = 0$ and $\rho \le \phi$, then the price of bonds, Q, is the same as in the economy without government intervention. This is because reserves with $i = 0$ are perfect substitutes of short-term capital and the reserve requirement is not binding as long as $\rho \le \phi$. Increasing the interest on reserves to above 0 or tightening the reserve requirement, setting $\rho > \phi$, have the effect of increasing the price of bonds above the laissez-faire level. As a consequence, these policies support the provision of liquidity-risk insurance.

Lemma 3. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with a reserve requirement and interest on reserves satisfies equations (8), (10) and*
$$\frac{C_2(1)}{C_1(0)} = \frac{R}{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{37}$$

Proof. Please refer to appendix A. □

Comparing the system of equations in lemma 3 with the first-best efficient allocation in definition 1, it is obvious that the government can make the economy fully efficient using its policy instruments. It is interesting to look at the combinations of interest on reserves and reserve requirement that achieve this, as this gives an insight in the relationship between the two policy instruments.

Proposition 2. *Define $h(i,\rho) = 0$ as the locus of policies with $i \geq 0$ and $\rho \in [0,1]$ that implement the first-best efficient allocation in the hidden-trade Diamond-Dybvig economy. We have that:*

1. *There exist policies (i,ρ) with $i \geq 0$ and $\rho \in [0,1]$ such that $h(i,\rho) = 0$.*
2. *There exists $\hat{\rho} \in [0,1]$ such that $h(0,\hat{\rho}) = 0$. $\hat{\rho} > \phi$.*
3. *There exists $\hat{i} \geq 0$ such that $h(\hat{i},0) = 0$. $\hat{i} > 0$.*
4. *On the domain $i \geq 0$ and $\rho \in [0,1]$, $\left.\frac{\mathrm{d}\rho}{\mathrm{d}i}\right|_{h(i,\rho)=0} < 0$.*

From the perspective of the government, interest on reserves and minimum reserve ratios are perfectly substitutable policies. If the government pays zero interest on reserves, there is a level of the reserve requirement that implements full efficiency. Conversely, in the absence of a reserve requirement, the government can implement optimality by paying a sufficiently high interest rate. Even only focusing on policies that use both instruments, if the reserve requirement is reduced, it is always possible to implement the optimal allocation by increasing the interest on reserves, and viceversa. In other words, it is unnecessary for the government to have both policy instruments at its disposal. It could forgo the use of either interest on reserves or the minimum reserve ratio and achieve the same level of welfare in the economy.

In this section, I replicated Farhi et al. (2009)'s result, according to which a minimum reserve ratio obtains the first-best allocation in the hidden-trade Diamond-Dybvig model. In addition, I showed that a policy of paying interest on reserves can equally implement full efficiency. In the next section, I extend the model making the level of intermediation of savings endogenous. I show that, if consumers can decide how much of their savings to deposit and how much to invest directly, the two policy instruments have different effects. It turns out that a reserve requirement is ineffective at promoting liquidity-risk insurance, because it leads to financial disintermediation. On the other hand, with interest on reserves the government can still implement the first best.

5 Endogenous Intermediation

As shown in the previous section, a reserve requirement or the payment of positive interest on bank reserves can equally implement the first-best allocation in the hidden-trade Diamond-Dybvig model. The fundamental problem that these policy measures solve is insufficient investment in short-term assets. The two policies fix this problem in different ways. A reserve requirement is a mandate for banks to hold a minimum amount of short-term assets and interest on reserves provides an incentive to hold more short-term assets. In effect, the two policies are equivalent: they make banks hold more short-term assets. Indeed, from the policymaker's prospective the policies are perfectly substitutable.

In this section, I show that the equivalence of interest on reserves and reserve requirement depends on an assumption. In the hidden-trade Diamond-Dybvig model, consumers must invest their entire endowment in bank deposits. They cannot invest directly in capital assets at time 0. In other words, full intermediation of consumers' savings through the banking system is imposed exogenously. I relax this assumption allowing consumers to choose the level of intermediation of their savings.

Compared to the hidden-trade Diamond-Dybvig model, the consumer's time-0 portfolio decision becomes more complicated. In addition to deposits $D_{j,k}$, she can choose to save directly in long-term capital $K_{j,L}$, as shown in the time-0 budget constraint:

$$\int_0^1 D_{j,k}\,\mathrm{d}k + K_{j,L} = E - T. \tag{38}$$

T are lumpsum taxes levied by the government. I restrict the consumer's portfolio allocation to long-term capital and bank deposits for simplicity. The same results would go through if I allowed the consumer to also hold short-term capital and reserves directly. This is because bank deposits are always a good short-term investment relative to alternatives.

The consumer's other two flow budget constraints are given by equation (13) and

$$C_{j,2}(\theta_j) = \int_0^1 (1+d_{k,1})\cdot[(1+d_{k,0})\cdot D_{j,k} - W_{j,k}(\theta_j)]\,\mathrm{d}k + S_j(\theta_j) + K_{j,L}. \tag{39}$$

I derive the consumer's demand for deposits from the optimisation problem. The key novelty in this version of the model is that consumers have an outside option: they also can hold long-term capital. So, the consumer will only hold deposits if she does not strictly prefer capital assets. Hence, in order to write the demand for deposits, I need to define a valuation function for long-term capital, which allows me to compare the benefits of investing in a bank deposit, as given by equation (17), with the benefits of direct capital investment. The value expressed in expected utility terms of a unit of the long-term asset at time 0 for a consumer j is given by:

$$V_{j,L} \equiv \phi\cdot u'[C_{j,1}(0)]\cdot R\cdot Q + (1-\phi)\cdot u'[C_{j,2}(1)]\cdot R. \tag{40}$$

Using equations (17) and (40), I can write the demand correspondence for deposits as

$$D_{j,k}(d_{k,0},d_{k,1}) = \begin{cases} E-T & \text{if } V_j(d_{k,0},d_{k,1}) > \max_{z\neq k}\left\{V_j(d_{z,0},d_{z,1}), V_{j,L}\right\}, \\ [0,E-T] & \text{if } V_j(d_{k,0},d_{k,1}) = \max_{z\neq k}\left\{V_j(d_{z,0},d_{z,1}), V_{j,L}\right\}, \\ 0 & \text{if } V_j(d_{k,0},d_{k,1}) < \max_{z\neq k}\left\{V_j(d_{z,0},d_{z,1}), V_{j,L}\right\}. \end{cases} \tag{41}$$

A consumer decides to hold all of her wealth in a deposit account if that is strictly the best investment for her. If another investment strictly dominates the deposit account, then the consumer will choose not to hold any of her wealth in the bank deposit. The consumer holds a mixed portfolio if she is indifferent between assets.

The definition of equilibrium in the hidden-trade Diamond-Dybvig model with endogenous intermediation is as follows.

Definition 4. *Given policy instruments $i \geq 0$ and $\rho \in [0,1]$, the equilibrium consists of a vector of quantities* $\left\{D_{j,k}, K_{j,L}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j), B_k, K_{k,S}, K_{k,L}, K_{g,S}, T, B_g\right\}_{(j,k,\theta_j)}$ *and a vector of prices* $\left(Q, \{d_{k,0}, d_{k,1}\}_{k\in[0,1]}\right)$ *such that:*

1. *Consumer i chooses quantities* $\left\{D_{j,k}, K_{j,L}, S_j(\theta_j), C_{j,1}(\theta_j), C_{j,2}(\theta_j), W_{j,k}(\theta_j)\right\}_{(k,\theta_j)}$ *to maximise (11) subject to constraints (38), (13), (39),*

$$(D_{j,k}, K_{j,L}) \geq 0 \quad \forall k \tag{42}$$

   and (15).

2. *Bank k chooses quantities* $\left(B_k, K_{k,S}, K_{k,L}, \{D_{j,k}, W_{j,k}(\theta_j)\}_{(j,\theta_j)}\right)$ *and a deposit contract given by* $(d_{k,0}, d_{k,1})$ *to maximise profits* Π_k *subject to constraints (16), (41), (28), (30), (31), (21) and (32).*

3. $(K_{g,S}, T, B_g)$ *are such that the government's budget constraints (26) and (27) hold.*

4. *The bond market clears:* $\int_0^1 S_j(\theta_j)\,\mathrm{d}j = 0$.

5. *The market for reserves clears:* $\int_0^1 B_k\,\mathrm{d}k = B_g$.

Policies are effective at promoting liquidity-risk insurance insofar as they push up the price of bonds at time 1, Q. A low level of Q represents an incentive for late-type consumers to withdraw deposits early and reinvest. This incentive curtails banks' ability to provide liquidity-risk insurance, because it makes it impossible to separate early types, who have liquidity needs, from late types.

When consumers are allowed to disintermediate, I find that a reserve requirement, regardless of the level at which it is set, has no effect at all on price Q. Only a policy of paying interest on reserves can put upward pressure on Q and therefore reduce the return from investing in the bonds market at time 1.

Lemma 4. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, a reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1+i}{R}. \tag{43}$$

and the prevailing deposit contract is given by:

$$d_0 = i, \tag{44}$$

$$1 + d_1 = \frac{R}{1+i}. \tag{45}$$

Proof. Please refer to appendix A. □

In the economy without government intervention, there is too little investment in short-term assets. Reserve requirements and interest on reserves attempt to fix this inefficiency in different ways, the former forces banks to invest more in short-term assets and the latter provides an incentive to invest more in short-term assets. However, in a model where intermediation is endogenous and therefore consumers react to an effective taxation of bank deposits by depositing less, a reserve requirement only has the effect of prodding consumers to disintermediate and invest directly in long-term assets. As a consequence, the reserve requirement does not change the economy's portfolio of short-term and long-term investment. On the other hand, interest on reserves provides an incentive to invest in short-term assets that succeeds at shifting the economy's investment portfolio towards short-term assets. In conclusion, interest on reserves promotes liquidity-risk insurance, while a reserve requirement does not.

Lemma 5. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, reserve requirements and interest on reserves satisfies equations (8), (10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{1+i}. \tag{46}$$

Proof. Please refer to appendix A. □

Clearly, by using the interest on reserves the government can implement the efficient allocation.

Proposition 3. *For any $\rho \in [0,1]$, there exists a unique optimal interest on reserves $i = i^* > 0$ that implements the first-best allocation in the equilibrium of the hidden-trade Diamond-Dybvig model with endogenous intermediation.*

While the reserve requirement does not have an impact on risk sharing in equilibrium, interest on reserves promotes liquidity-risk insurance in the economy. The level of interest on reserves that implements full efficiency in the economy is always strictly positive.

To study in greater detail how the optimal interest on reserves, i^*, varies with the model's parameters, I choose to restrict the utility function to feature constant relative risk aversion.

Proposition 4. *Consider utility function $u(C) = \frac{C^{1-\gamma}-1}{1-\gamma}$, with $\gamma \geq 1$ coefficient of relative risk aversion. The resulting formula for the optimal interest on reserves is given by*

$$\ln(1+i^*) = \ln(R) - \frac{1}{\gamma} \cdot \ln(\beta \cdot R). \tag{47}$$

Properties of the optimal interest on reserves are that:

1. *The optimal interest on reserves is non-decreasing in the return on long-term investments R, with $\frac{\partial \ln(1+i^*)}{\partial \ln(R)} = \frac{\gamma-1}{\gamma} \geq 0$.*

2. *The optimal interest on reserves is strictly decreasing in the consumers' discount factor β, with $\frac{\partial \ln(1+i^*)}{\partial \ln(\beta)} = -\frac{1}{\gamma} < 0$.*

3. *The optimal interest on reserves is increasing in the consumers' coefficient of relative risk aversion at a decreasing rate γ, with* $\frac{\partial \ln(1+i^*)}{\partial \gamma} = \frac{\ln(\beta \cdot R)}{\gamma^2} > 0$ *and* $\frac{\partial^2 \ln(1+i^*)}{\partial \gamma^2} = -2 \cdot \frac{\ln(\beta \cdot R)}{\gamma^3} < 0$.

Since the rationale for paying positive interest on reserves is to improve liquidity-risk sharing, it stands to reason that parametrisations that increase demand for such insurance, i.e. a low discount factor β and a high coefficient of relative risk aversion γ, call for a higher interest on reserves. A higher return on long-term assets R implies a higher optimal interest on reserves, too. Nonetheless, an optimally set interest rate on reserves is never greater than the long-term rate of return in the economy: $1 + i^*$ tends to R as relative risk aversion tends to infinity.

A reserve requirement, the policy measure proposed by Farhi et al. (2009) to deal with insufficient liquidity-risk insurance, is ineffective in a model where consumers can choose to invest directly in capital in addition to depositing their wealth in banks. It causes consumers to disintermediate and invest directly in long-term assets, since banks are effectively taxed by the obligation to hold a suboptimal asset portfolio and this lowers the returns of bank deposits.

Interest on reserves promotes liquidity-risk insurance in a Diamond-Dybig model in which financial markets coexist alongside the banking sector and the level of financial intermediation is endogenous. By means of an appropriately set interest on reserves, the government can implement the first-best allocation in equilibrium.

6 Fiscal Consequences

Offering a positive interest on reserves requires the government to levy taxes, because a positive interest on reserves is a subsidy on short-term assets in this model. Using the government's budget constraints (26) and (27), we can see that

$$T = i \cdot B_g. \tag{48}$$

Notice that, since i is weakly positive, I do not consider negative taxes, i.e. transfers, in this section.[3]

In the previous sections, I assumed that taxes do not distort any of the decisions made in the economy. A more serious analysis of the fiscal consequences of policies should allow for taxes to reduce the incentives to produce and thus generate a deadweight loss. Accordingly, I postulate that the endowment depends inversely on the level of taxation, defining a function as follows.

Definition 5. *Define a twice-continously differentiable function $E : \mathbb{R} \to \mathbb{R}_+$ that maps taxes, T, into endowment, E. The function is characterised by:*

1. $\frac{\partial E(T)}{\partial T} \leq 0 \quad \forall\, T,$
2. $\frac{\partial^2 E(T)}{\partial T^2} \leq 0 \quad \forall\, T.$

[3] At negative values of i, there would be no demand for reserves. Hence, taxes would be zero.

The definition of the endowment function nests the case of non-distortionary taxation, which is the assumption maintained so far in the paper. Non-distortionary taxation corresponds to a function with first-order derivative equal to zero at all levels of taxation.

In the rest of this section, I study optimal monetary policy in a model with deadweight loss due to taxation. I organise the section in two subsections, the first dedicated to the hidden-trade Diamond-Dybvig model with exogenous intermediation and the second to the model with endogenous intermediation.

6.1 Exogenous Intermediation

In the hidden-trade Diamond-Dybvig model with exogenous intermediation, consumers can only save through the banking system at time 0. The model is described in section 4.2 of this paper. In this analysis, I let the endowment depend on the level of taxes according to the function in definition 5.

As according to equation (48), the level of taxation that the government needs depends on the demand for reserves and on the interest paid on them. In turn, the demand for reserves is increasing in the endowment, in the probability ϕ of being hit by the liquidity shock and in the reserve requirement ρ. In equilibrium, taxes are given by:

$$T = \frac{\max\{\phi, \rho\} \cdot i}{1 + \max\{\phi, \rho\} \cdot i} \cdot E(T) \tag{49}$$

While a strictly positive interest rate per se implies a strictly positive amount of taxation, any level of the reserve requirement is compatible with zero taxes as long as the interest on reserves is zero.

Proposition 5. *Consider $E(T)$ with $\frac{\partial E(T)}{\partial T} < 0$ for all T. There exists a unique policy (i, ρ) with $i = 0$ and $\rho = \rho^* \in (\phi, 1)$ that implements the first-best efficient allocation in the equilibrium of the hidden-trade Diamond-Dybvig model with exogenous intermediation and distortionary taxation.*

In the case, illustrated in section 4.2, of non-distortionary taxation, I found that interest on reserves and a reserve requirement are perfectly substitutable policy instruments from the prospective of welfare-maximising policy makers.

Distortionary taxation breaks the equivalence of the policy instruments in this model. Welfare-maximising policy makers should avoid policies that require taxes to be sustained. Since interest on reserves makes taxation necessary whereas a reserve requirement does not, a reserve requirement is a superior instrument to facilitate liquidity-risk insurance and implement the first-best efficient allocation.

6.2 Endogenous Intermediation

In the hidden-trade Diamond-Dybvig model with endogenous intermediation, consumers can save at time 0 through financial intermediaries or directly in financial markets. The model is described in section 5 of this paper. In this subsection, taxes generate a deadweight loss by reducing the endowment, as described in definition 5.

Unlike in the model with exogenous intermediation, demand for bank reserves does not depend on the reserve requirement when consumers can decide how much of their savings to deposit. An increase in the reserve requirement makes banks hold more of their assets as reserves but also makes consumers hold less of the their wealth in bank deposits. It turns out that the two effects cancel out. Thus, the level of taxation is given by

$$T = \frac{\phi \cdot i}{1 + \phi \cdot i} \cdot E(T). \tag{50}$$

A reserve requirement does not require taxation to be imposed. However, in this model a reserve requirement also has no effect on the overall level of investment in short-term assets. Therefore, it is useless in terms of improving liquidity-risk insurance in the economy. To this aim, the government can use interest on reserves effectively. It faces a trade-off in doing so though, because interest on reserves requires taxation.

Since policy makers cannot in general obtain the first-best efficient allocation, I define the optimal monetary policy problem.

Definition 6. *Optimal policy (i, ρ) in the hidden-trade Diamond-Dybvig model with endogenous intermediation and distortionary taxation maximises aggregate welfare, given by (11), subject to variables being determined in equilibrium, as described in definition 4, with additional endogenous variable E determined by*

$$E = E(T). \tag{51}$$

A welfare-maximising policy maker would choose (i, ρ) to solve the optimal policy problem, as defined above. The result of such optimisation follows.

Lemma 6. *Optimal policy (i, ρ) in the hidden-trade Diamond-Dybvig model with endogenous intermediation and distortionary taxation implements the allocation given by*

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = \beta \cdot R \cdot \min \left\{ R^{\gamma[C_2(1)]-1}, 1 - \frac{\left\{ \phi \cdot \left(\frac{R}{1+i} \right)^{\gamma[C_2(1)]} + 1 - \phi \right\} \cdot E'(T)}{(1-\phi) \cdot \{1 + [1 - E'(T)] \cdot \phi \cdot i\}} \right\}, \tag{52}$$

$$\phi \cdot C_1(0) + (1-\phi) \cdot \frac{C_2(1)}{R} = E(T), \tag{53}$$

and equation (46).

Proof. Please refer to appendix A. □

From equation (52), we can see that, if taxation is distortionary, a welfare-maximising government reduces its use of interest on reserves to support liquidity-risk insurance.

Proposition 6. *If $E'(T) < 0$ for all T, then the optimal allocation in the hidden-trade Diamond-Dybvig model with endogenous intermediation is not first-best efficient.*

In the model with endogenous intermediation, the only way for the government to promote liquidity-risk insurance, which otherwise is inefficiently low, is to pay interest on reserves. However, interest on reserves is a policy that needs to be financed with taxes. Hence, if taxation creates distortions, the government should implement a less than first-best level of liquidity-risk insurance in order to reduce the inefficiencies created by taxes.

7 Conclusion

The existence of financial markets in the Diamond-Dybvig model reduces the scope for banks to provide liquidity-risk insurance, as shown by Jacklin (1987). The equilibrium allocation features inefficiently low aggregate investment in short-term assets.

Farhi et al. (2009) conclude that the inefficiency can be fixed by imposing a reserve requirement on the banking system. This forces banks to invest more in short-term assets and therefore increases the economy-wide level of investment in short-term assets. My first result is that within the Diamond-Dybvig framework adopted by Farhi et al. (2009) a policy of paying positive interest on reserves can achieve the same allocation as a reserve requirement. What a reserve requirement achieves with regulation, interest on reserves achieves by providing an incentive to invest more in short-term assets.

The Diamond-Dybvig model restricts consumers to investing their wealth solely in bank deposits. In the central section of this paper, I relax this assumption and allow consumers to also invest directly in financial markets. In this more general set-up, I find that a reserve requirement is unable to increase aggregate investment in short-term assets. While it does increase the share of the banks' asset portfolio invested in short-term assets, it also makes consumers decide to hold fewer deposits, since it acts as a tax on bank deposits. It turns out that the two effects, the increase in the share of bank investment in short-term assets and consumer disintermediation, cancel out. Hence, a reserve requirement has no effect on the aggregate level of short-term assets. On the other hand, the incentive to invest more in short-term assets provided by the payment of interest on bank reserves effectively increases aggregate investment in short-term assets, as it does not discourage financial intermediation.

The paper's key conclusion is that interest on reserves is effective at promoting liquidity-risk insurance. It is a superior policy to the setting of a reserve requirement, because it does not lead to financial disintermediation.

In the last section of the paper, I study the fiscal consequences of interest on reserves. Paying interest on reserves is costly for the government and must therefore be financed with taxation. I postulate an inverse relation between the endowment and taxes, justified by the presence of a deadweight loss associated with distortionary taxes. If consumers cannot disintermediate their wealth, then the government finds it optimal to impose a reserve requirement and not to pay interest on reserves, because a reserve requirement does not require taxes. On the other hand, if intermediation is endogenous, as discussed above, a reserve requirement is ineffective at promoting liquidity-risk insurance. Thus, it is optimal for the government to pay interest on reserves. However, payment of interest on reserves involves a policy trade-off for the government: a higher interest on reserves improves liquidity-risk insurance but reduces output via the distorting effects of taxes. Ultimately, I find that the more distorting taxes are, the lower the interest on reserves should be.

Studying optimal interest-rate setting and its relationship with the payment of interest on reserves and liquidity would be interesting. However, this paper's model is too simple to study interest-rate setting. Crucially, there is no meaningful saving decision taken by consumers at time 0. The necessary elements to study interest-rate setting in a Diamond-Dybvig model can be found in Porcellacchia (2018). I leave this extension to future work.

Since in reality banks do perform maturity transformation and financial markets exist, the merit of interest on reserves should be studied in a setting where banks have scope to provide liquidity-risk insurance despite their coexistence with financial markets. Diamond (1997) is an important example of a model with such characteristics.

The fragility of deposit contracts is a key result of the Diamond-Dybvig model, which this paper treats as orthogonal to the monetary policies analysed. This is because a financial crisis is the model's bad equilibrium, which simple deposit insurance can prevent from ever taking place. Once deposits are insured, there is no relationship between monetary policy and bank runs. Global games, developed in Carlsson and van Damme (1993), deliver a unique equilibrium in which financial crises take place with non-zero probability. Hence, global games hold promise to shed light on the relationship between monetary-policy instruments, such as interest on reserves, and the probability of financial crises.

References

Allen, Franklin, Elena Carletti, and Douglas Gale. 2014. "Money, Financial Stability and Efficiency." *Journal of Economic Theory*, 149(C): 100–127.

Allen, Franklin, and Douglas Gale. 2004. "Financial Intermediaries and Markets." *Econometrica*, 72(4): 1023–1061.

Antinolfi, Gaetano, and Suraj Prasad. 2008. "Commitment, Banks and Markets." *Journal of Monetary Economics*, 55(2): 265–277.

Canzoneri, Matthew, Robert Cumby, and Behzad Diba. 2017. "Should the Federal Reserve Pay Competitive Interest on Reserves?." *Journal of Money, Credit and Banking*, 49(4): 663–693.

Carlsson, Hans, and Eric van Damme. 1993. "Global Games and Equilibrium Selection." *Econometrica*, 61(5): 989–1018.

Cochrane, John H. 2014. "Monetary Policy with Interest on Reserves." *Journal of Economic Dynamics and Control*, 49(C): 74–108.

Cúrdia, Vasco, and Michael Woodford. 2011. "The Central-Bank Balance Sheet as an Instrument of Monetary Policy." *Journal of Monetary Economics*, 58(1): 54–79.

Diamond, Douglas W. 1997. "Liquidity, Banks, and Markets." *Journal of Political Economy*, 105(5): 928–956.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

Ennis, Huberto M. 2014. "A Simple General Equilibrium Model of Large Excess Reserves." Working Paper 14-14, Federal Reserve Bank of Richmond.

Ennis, Huberto M, and John A Weinberg. 2007. "Interest on Reserves and Daylight Credit." *Economic Quarterly*, 93(2): 111–142.

Farhi, Emmanuel, Mikhail Golosov, and Aleh Tsyvinski. 2009. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies*, 76(3): 973–992.

Goldstein, Itay, and Ady Pauzner. 2005. "Demand-Deposit Contracts and the Probability of Bank Runs." *Journal of Finance*, 60(3): 1293–1327.

Hasman, Augusto, Margarita Samartín, and Jos van Bommel. 2014. "Financial Intermediation in an Overlapping Generations Model with Transaction Costs." *Journal of Economic Dynamics and Control*, 45(C): 111–125.

Jacklin, Charles J. 1987. "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade*. eds. by Edward C. Prescott, and Neil Wallace: University of Minnesota Press, 26–47.

Kashyap, Anil K., and Jeremy C. Stein. 2012. "The Optimal Conduct of Monetary Policy with Interest on Reserves." *American Economic Journal: Macroeconomics*, 4(1): 266–282.

Panetti, Ettore. 2017. "A Theory of Bank Illiquidity and Default with Hidden Trades." *Review of Finance*, 21(3): 1123–1157.

Porcellacchia, Davide. 2018. "Optimal Negative Interest on Reserves: the Maturity-Transformation Channel of Monetary Policy Transmission."

Reis, Ricardo. 2016. "Funding Quantitative Easing to Target Inflation." Discussion Papers 1626, Centre for Macroeconomics (CFM).

Rochet, Jean-Charles, and Xavier Vives. 2004. "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?." *Journal of the European Economic Association*, 2(6): 1116–1147.

Sargent, Thomas, and Neil Wallace. 1985. "Interest on Reserves." *Journal of Monetary Economics*, 15(3): 279–290.

von Thadden, Ernst-Ludwig. 1999. "Liquidity Creation Through Banks and Markets: Multiple Insurance and Limited Market Access." *European Economic Review*, 43(4-6): 991–1006.

A Proofs

Lemma 1. *In the equilibrium of the Diamond-Dybvig economy with hidden trades, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1}{R} \tag{23}$$

and the prevailing deposit contract is given by

$$(d_0, d_1) = (0, R-1). \tag{24}$$

Proof. Define $\zeta = \frac{\phi \cdot W(0)+(1-\phi)\cdot W(1)}{(1+d_0)\cdot E}$, the share of deposits withdrawn at time 1. The market clearing condition in the hidden bond market

$$\phi \cdot S(0) + (1-\phi) \cdot S(1) = 0, \tag{54}$$

the consumer's budget constraints (12), (13) and (14), and the fact that $C_2(0) = C_1(1) = 0$ imply that in equilibrium

$$\zeta = \phi. \tag{55}$$

By the withdrawing behaviour equation (16), then it must be that in equilibrium $1 + d_1 = Q^{-1}$. Otherwise, all consumers would either not withdraw at all at time 1, so that $\zeta = 0$, or withdraw all their deposits, so that $\zeta = 1$.

Banks offer competitively deposit contracts (d_0, d_1), which imply a withdrawing behaviour ζ. Banks make zero profits because of Bertrand-style competition. By the bank's budget constraints (19), (20) and (21), this implies that

$$1 + d_0 = \frac{R \cdot Q}{\zeta \cdot R \cdot Q + 1 - \zeta}. \tag{56}$$

A bank can deviate and earn strictly positive profits unless in equilibrium the deposit contract on offer solves

$$\max_{\zeta \in [0,1]} \frac{1}{Q} \cdot \left\{ \phi \cdot u'[C_1(0)] + (1-\phi) \cdot \beta u'[C_2(1)] \right\} \cdot \frac{R \cdot Q}{\zeta \cdot R \cdot Q + 1 - \zeta}. \tag{57}$$

Note that banks can choose the timing of depositor withdrawal because $1 + d_1 = Q^{-1}$ makes consumers indifferent.

The solution of the above is given by the deposit contract which gives

$$\zeta = \begin{cases} 0 & \text{if } R \cdot Q > 1, \\ [0,1] & \text{if } R \cdot Q = 1, \\ 1 & \text{if } R \cdot Q < 1. \end{cases} \tag{58}$$

Since, as shown above, $\zeta = \phi$ in equilibrium, we have that

$$Q = \frac{1}{R}. \tag{59}$$

□

Proposition 1. *The equilibrium allocation of the Diamond-Dybvig model with hidden trades satisfies equations (8), (10) and*

$$\frac{C_2(1)}{C_1(0)} = R. \tag{25}$$

It is not first best.

Proof. With the budget constraints (12), (13), (14) and the equilibrium prices given in lemma 1, we find that the equilibrium allocation satisfies equations (8), (10) and (25). Now, given a coefficient of relative risk aversion larger than 1 as implied by restriction (2) on the felicity function, I show that the equilibrium allocation is not first best.
By Euler's homogeneous function theorem, function $u'(C)$ is homogenous of degree $\frac{u''(C)\cdot C}{u'(C)} < -1$ for all C. This implies that

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = R^{-\frac{u''(R\cdot E)\cdot R\cdot E}{u'(R\cdot E)}} > R. \tag{60}$$

Thus, the allocation is not first-best efficient. □

Lemma 2. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}{R}. \tag{34}$$

and the prevailing deposit contract is given by:

$$1+d_0 = \max\left\{1, \frac{\rho}{\phi}\right\}\cdot(1+i), \tag{35}$$

$$1+d_1 = \frac{R}{(1+i)\cdot\max\left\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\right\}}. \tag{36}$$

Proof. Define $\zeta = \frac{\phi\cdot W(0)+(1-\phi)\cdot W(1)}{(1+d_0)\cdot E}$, the share of deposits withdrawn at time 1. The market clearing condition in the hidden bond market

$$\phi\cdot S(0) + (1-\phi)\cdot S(1) = 0, \tag{61}$$

the consumer's budget constraints (29), (13) and (14), and the fact that $C_2(0) = C_1(1) = 0$ imply that in equilibrium

$$\zeta = \phi. \tag{62}$$

By the withdrawing behaviour equation (16), then it must be that in equilibrium $1+d_1 = Q^{-1}$. Otherwise, all consumers would either not withdraw at all at time 1, so that $\zeta = 0$, or withdraw all their deposits, so that $\zeta = 1$.

Banks offer competitively deposit contracts (d_0, d_1), which imply a withdrawing behaviour ζ. Banks make zero profits because of Bertrand-style competition. By the budget constraints (30), (31) and (21) and by the reserve requirement (28), this implies that

$$1 + d_0 = \min\left\{\frac{R \cdot Q \cdot (1+i)}{\zeta \cdot R \cdot Q + (1-\zeta) \cdot (1+i)}, \frac{1-\rho}{1-\zeta} \cdot R \cdot Q\right\}. \tag{63}$$

A bank can deviate and earn strictly positive profits unless in equilibrium the deposit contract on offer solves

$$\max_{\zeta \in [0,1]} \frac{1}{Q} \cdot \left\{\phi \cdot u'[C_1(0)] + (1-\phi) \cdot \beta u'[C_2(1)]\right\} \cdot \min\left\{\frac{R \cdot Q \cdot (1+i)}{\zeta \cdot R \cdot Q + (1-\zeta) \cdot (1+i)}, \frac{1-\rho}{1-\zeta} \cdot R \cdot Q\right\}. \tag{64}$$

Note that banks can choose the timing of depositor withdrawal because $1 + d_1 = Q^{-1}$ makes consumers indifferent.
The solution of the above is given by the deposit contract which gives

$$\zeta = \begin{cases} \frac{\rho \cdot (1+i)}{\rho \cdot (1+i) + (1-\rho) \cdot R \cdot Q} & \text{if } R \cdot Q > 1 + i, \\ [\rho, 1] & \text{if } R \cdot Q = 1 + i, \\ 1 & \text{if } R \cdot Q < 1 + i. \end{cases} \tag{65}$$

Since, as shown above, $\zeta = \phi$ in equilibrium, we have that

$$Q = \frac{(1+i) \cdot \max\left\{1, \frac{(1-\phi) \cdot \rho}{\phi \cdot (1-\rho)}\right\}}{R}. \tag{66}$$

□

Lemma 3. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with a reserve requirement and interest on reserves satisfies equations (8), (10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{(1+i) \cdot \max\left\{1, \frac{(1-\phi) \cdot \rho}{\phi \cdot (1-\rho)}\right\}}. \tag{37}$$

Proof. With the budget constraints (29), (13), (14) and the equilibrium prices given in lemma 2, we find that the equilibrium allocation satisfies equations (8), (10) and (37).

□

Lemma 4. *In the equilibrium of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, a reserve requirement and interest on reserves, the price of bonds traded at time 1 in the anonymous market is given by*

$$Q = \frac{1+i}{R}. \tag{43}$$

and the prevailing deposit contract is given by:

$$d_0 = i, \tag{44}$$

$$1 + d_1 = \frac{R}{1+i}. \tag{45}$$

Proof. Define $\zeta = \frac{\phi \cdot W(0) + (1-\phi) \cdot W(1)}{(1+d_0) \cdot D}$, the share of deposits withdrawn at time 1. The market clearing condition in the hidden bond market

$$\phi \cdot S(0) + (1-\phi) \cdot S(1) = 0, \tag{67}$$

the consumer's budget constraints (29), (13) and (14), and the fact that $C_2(0) = C_1(1) = 0$ imply that in equilibrium

$$\phi \cdot \int_{j=0}^{1} K_{j,L} \, \mathrm{d}j = (1+d_0) \cdot (1+d_1) \cdot D \cdot (\zeta - \phi). \tag{68}$$

By non-negativity of deposits and capital holdings, from this equation we can conclude that in equilibrium $\zeta \geq \phi$. Moreover, if in equilibrium $\zeta > \phi$, then $D > 0$ and $\int_{j=0}^{1} K_{j,L} \, \mathrm{d}j > 0$. And, if $\zeta = \phi$, then $D > 0$ and $\int_{j=0}^{1} K_{j,L} \, \mathrm{d}j = 0$.

Focus on the case $\zeta > \phi$. This requires $V(d_0, d_1) = V_K$, so that consumers are indifferent between deposits and direct capital holdings and hold both in equilibrium. We find that

$$\begin{aligned} V(d_0,d_1) - V_K = & \frac{1}{Q} \cdot \Big\{ \phi \cdot u'[C_1(0)] + (1-\phi) \cdot \beta u'[C_2(1)] \Big\} \cdot \frac{\zeta}{\zeta \cdot R \cdot Q + (1-\zeta) \cdot (1+i)} \cdot \\ & \cdot \Big(1 + i - R \cdot Q \Big). \end{aligned} \tag{69}$$

Hence, in equilibrium if $\zeta > \phi$, then $Q = \frac{1+i}{R}$, $d_0 = i$ and $1 + d_1 = \frac{R}{1+i}$.

Now consider $\zeta = \phi$ and $\int_{j=0}^{1} K_{j,L} \, \mathrm{d}j = 0$. The equilibrium is identical to the one described by lemma 2. In such equilibrium,

$$Q = \frac{(1+i) \cdot \max\Big\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\Big\}}{R} \tag{70}$$

and

$$(1+d_0, 1+d_1) = \left(\max\Big\{1, \frac{\rho}{\phi}\Big\} \cdot (1+i), \frac{R}{(1+i) \cdot \max\Big\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\Big\}} \right). \tag{71}$$

If we verify whether consumers have an incentive to invest one unit of their endowment in directly in capital rather than in their deposit, we find that

$$\begin{aligned} V(d_0,d_1) - V_K = & \frac{1}{Q} \cdot \Big\{ \phi \cdot u'[C_1(0)] + (1-\phi) \cdot \beta u'[C_2(1)] \Big\} \cdot \frac{\phi \cdot (1+i)}{\phi \cdot R \cdot Q + (1-\phi) \cdot (1+i)} \cdot \\ & \cdot \left(1 - \max\Big\{1, \frac{(1-\phi)\cdot\rho}{\phi\cdot(1-\rho)}\Big\} \right) \leq 0. \end{aligned} \tag{72}$$

Hence, whenever $\rho > \phi$ consumers have a strict incentive to hold capital directly rather than deposits. The equilibrium features $\zeta = \phi$ if and only if $\rho \leq \phi$. Also in this case we have that $Q = \frac{1+i}{R}$.

For any policy (ρ, i), in equilibrium $Q = \frac{1+i}{R}$. □

Lemma 5. *The equilibrium allocation of the hidden-trade Diamond-Dybvig economy with endogenous intermediation, reserve requirements and interest on reserves satisfies equations (8), (10) and*

$$\frac{C_2(1)}{C_1(0)} = \frac{R}{1+i}. \tag{46}$$

Proof. With the budget constraints (29), (13), (14) and the equilibrium prices given in lemma 4, we find that the equilibrium allocation satisfies equations (8), (10) and (46). □

Lemma 6. *Optimal policy (i, ρ) in the hidden-trade Diamond-Dybvig model with endogenous intermediation and distortionary taxation implements the allocation given by*

$$\frac{u'[C_1(0)]}{u'[C_2(1)]} = \beta \cdot R \cdot \min\left\{R^{\gamma[C_2(1)]-1}, 1 - \frac{\left\{\phi \cdot \left(\frac{R}{1+i}\right)^{\gamma[C_2(1)]} + 1 - \phi\right\} \cdot E'(T)}{(1-\phi) \cdot \{1 + [1 - E'(T)] \cdot \phi \cdot i\}}\right\}, \tag{52}$$

$$\phi \cdot C_1(0) + (1-\phi) \cdot \frac{C_2(1)}{R} = E(T), \tag{53}$$

and equation (46).

Proof. Equations (52), (53) and (46) are the first-order conditions of the maximisation of objective function (11) with respect to variables $(C_1(0), C_2(1), i, T)$ subject to equations that define the variables in equilibrium:

$$C_1(0) = \frac{1+i}{1+\phi \cdot i} \cdot E(T), \tag{73}$$

$$C_2(0) = \frac{R}{1+\phi \cdot i} \cdot E(T), \tag{74}$$

$$T = \frac{\phi \cdot i}{1+\phi \cdot i} \cdot E(T), \tag{75}$$

$$i \geq 0. \tag{76}$$

□