

# The Macroeconomics of Liquidity in Financial Intermediation<sup>a</sup>

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## Abstract

The liquidity premium on US Treasuries is positively correlated with credit spreads, including bank funding costs. To explain this fact, we develop a theory of endogenous bank fragility arising from a coordination friction among bank creditors and embed it in a macroeconomic model. Adverse shocks to bank net worth exacerbate the friction causing banks to lend less and demand more liquid assets, driving up both credit spreads and the liquidity premium. By mitigating the friction, expansions of public liquidity reduce spreads and boost output. Using high-frequency data and exploiting the lag between the auction and issuance of US Treasuries, we identify liquidity-supply shocks and confirm a positive causal effect of the liquidity premium on spreads.

**Keywords:** liquidity, bank-lending channel, bank runs.

**JEL Codes:** E40, E41, E44, E50, E51, G01, G21.

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# 1 Introduction

Disruptions to financial intermediation make credit more expensive and thereby harm the economy. This observation has motivated adding a specific banking friction to macroeconomic models. In their seminal contribution, [Gertler and Kiyotaki \(2010\)](#) introduce a problem of moral hazard between banks and their creditors. In consequence, banks' ability to fund themselves is limited by the value of their equity. The resulting leverage constraint leads to a powerful impact of bank net worth on macroeconomic outcomes through credit spreads. This explains the general observation of plummeting bank values, higher bank funding costs, and increased credit spreads in financial crises. However, this approach to banking is silent on why we see soaring demands for liquidity and hence liquidity premiums in times of financial stress.

Strong demand for liquid assets during banking crises is reflected in a heightened liquidity premium, defined as the difference between the 3-month general-collateral repo rate and the 3-month U.S. Treasury-bill rate.<sup>1</sup> [Figure 1](#) shows this for the 2007–8 financial crisis. Banks' funding costs, as measured by the difference between 3-month LIBOR and the 3-month repo rate, are also shown in the figure as an indicator of stress in the banking sector.<sup>2</sup> We observe a positive relationship over time between the liquidity premium and the bank funding spread, and [Figure 2](#) shows the positive correlation between these two variables.<sup>3</sup> Generally, banking stress and a scarcity of liquid assets go hand in hand.

In addition to the empirical link between liquidity and stress in the banking sector, policymakers typically respond to banking crises with expansions of liquidity, which are known to reduce the liquidity premium ([Krishnamurthy and Vissing-Jorgensen, 2012](#); [Nagel, 2016](#); [Krishnamurthy and Li, 2023](#)). However, evaluating policies such as quantitative easing requires understanding how their effect on the liquidity premium ultimately influences the supply of credit.<sup>4</sup> The facts documented here are consistent with a view that scarce liquidity impairs bank lending in times of stress, pointing to a channel through which a greater supply of public liquidity is beneficial.

Motivated by this, we do two things in this paper. First, we dig deeper into the empirical evidence and find that the liquidity premium has a positive *causal* effect on banks' funding costs. Second, we develop a theory of a novel financial friction based on the risk of a coordination failure among bank creditors. This theory explains why an abundance of liquid assets makes it easier for banks to fund themselves by reducing this risk and thereby lowering their funding costs. Since higher bank net worth also

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<sup>1</sup>This definition is standard in the literature ([Nagel, 2016](#)). See [section 2](#) for further discussion.

<sup>2</sup>[Figure 9](#) in [appendix A](#) zooms in on the recent period between 2019 and 2023.

<sup>3</sup>[Figure 10](#) shows that the positive correlation holds both in expansions and recessions. A scatterplot with data at monthly frequency rather than binned is available in the online appendix.

<sup>4</sup>There is a debate in the literature on the real effects of liquidity policies and the channels through which they operate ([Kuttner, 2018](#)).

Figure 1: 2007–8 financial crisis.

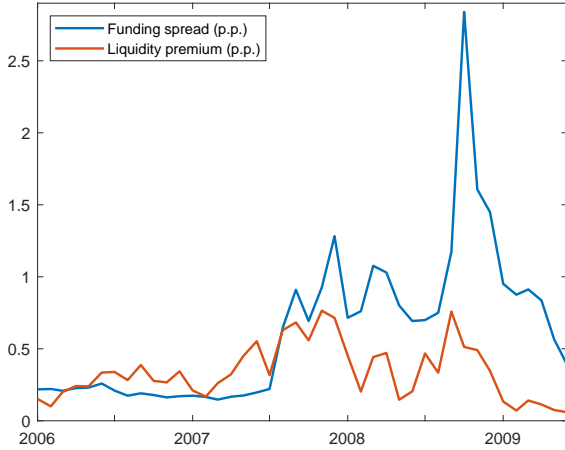
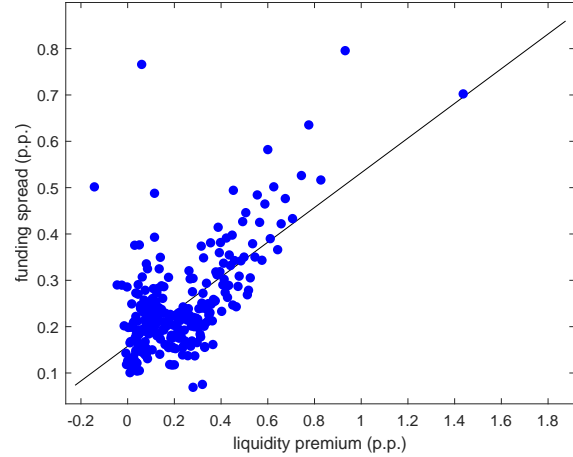


Figure 2: May 1991 – June 2023.



Note 1: The funding spread is 3-month (3M) LIBOR minus the 3M general-collateral (GC) repo rate. The liquidity premium is the 3M GC repo rate minus the 3M T-bill rate.

Note 2: US daily data. Figure 1 plots data at monthly frequency. Figure 2 plots a binned scatterplot with 300 quantile-based bins. Data sources are found in the online appendix.

mitigates the coordination friction, bank equity and holdings of liquid assets act as substitutes. Thus, when net worth is scarcer in financial crises, banks demand more liquidity in response, which explains why the liquidity premium rises in banking crises. It also implies policymakers can stabilize the economy by appropriately supplying liquid assets in response to shocks.

Finding exogenous variation in the liquidity premium is the key challenge in estimating its causal effect on the bank funding spread. Our strategy is to use the quantity of outstanding U.S. Treasuries as an instrument and perform the analysis at daily frequency. This instrument is strongly relevant to the liquidity premium. As for its validity, the quantity of Treasuries is predetermined at daily frequency because there is a lag of a few days from auction to issuance, the auction being the latest point at which the quantity issued may respond to events.<sup>5</sup> Moreover, we include as controls 80 lags of financial and economic variables available at daily frequency, such as the dollar exchange rate and the liquidity premium itself, to clean autocorrelation from the error term. This rules out endogeneity of the instrument due to confounding variables or reverse causality because an error term that contains only a serially uncorrelated daily shock cannot cause changes in a variable determined on an earlier day.

The result of the IV regression is a statistically significant positive effect of the liquidity premium on the bank funding spread. Quantitatively, a 1-basis-point increase in the liquidity premium causes the funding spread to increase by approximately 1 basis point. As a robustness check, we split the sample between expansions and recessions but find no evidence of a differently sized effect according to the state of the economy.

To explain why abundant liquid assets help banks fund themselves, we develop a model with a risk of coordination failure in the market for bank deposits ([Diamond](#)

<sup>5</sup>A related high-frequency approach to Treasury-market data is adopted in [Ray et al. \(2024\)](#).

and Dybvig, 1983).<sup>6</sup> Coordination failures are possible because of a core function of financial intermediation, maturity transformation, which results in maturity mismatch on banks' balance sheets.<sup>7</sup> Such coordination failures take the form of 'runs' by panicked creditors. These played a key role in the global financial crisis in 2007, the crisis of US money-market funds in 2020, and the 2023 regional banking crisis (Shin, 2009; Bernanke, 2010; Li et al., 2021; Choi et al., 2023).

In models of bank runs, depositors' strategic complementarity implies that under perfect information there are multiple equilibria. However, a deviation from common knowledge across depositors, which we introduce following the large literature on global games (Morris and Shin, 2003), leads to a unique equilibrium. Intuitively, without common knowledge, it is impossible for depositors to coordinate on arbitrary equilibria. In the resulting unique equilibrium, depositors demand a level of compensation that is commensurate to a bank's fragility, defined as the minimum fraction of depositors that must not run for the bank to survive. If a bank offers an insufficiently low interest rate on deposits, then depositors run even though the bank is solvent because they fear a run by other households. On the other hand, as long as the bank offers a sufficiently high deposit rate, no run takes place because no depositor has an incentive to start the run that they fear.

Bank fragility, at the heart of the coordination friction, is endogenous. It is a function of a bank's balance-sheet fundamentals. In particular, more levered banks and banks with fewer liquid assets as a share of total assets are more fragile, and consequently face higher funding costs.<sup>8</sup> In other words, the coordination friction results in a mapping from higher capital and liquidity ratios to a lower funding spread. In choosing these capital and liquidity ratios, banks trade off the higher returns from illiquid assets against the increased funding costs due to greater fragility. Interestingly, the analysis implies the possibility of bank runs imposes economic costs by reducing bank lending even when a run does not occur.

The coordination friction can be tractably embedded in a standard real business cycle model and the parameters governing its size can be calibrated from knowing the average liquidity premium, credit spread, and return on bank equity. Hence, we can study its role in the transmission of macroeconomic shocks quantitatively.

The friction amplifies the impact of shocks that affect banks' net worth. By making it more costly for banks to fund themselves, a reduction in net worth weakens the supply

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<sup>6</sup>We use 'banks' as a general label for financial intermediaries and 'deposits' for their short-term debt. The analysis applies more broadly to intermediaries that fund long-term assets with shorter-term liabilities.

<sup>7</sup>Unlimited deposit insurance would rule out coordination failures. However, in the period 1984–2023Q3 deposits made up 73% of banks' liabilities and only 62% of these were insured on average. These values were respectively 79% and 57% in 2023Q3 (Source: FDIC QBP).

<sup>8</sup>DeYoung et al. (2018) provide evidence that bank net worth and holdings of liquid assets are substitutable in giving banks access to funding.

of credit and lowers GDP. The friction raises the effect on output of capital-destruction shocks, commonly studied in the literature on financial crises, by about one third on impact. At longer horizons, amplification is greater. This persistence comes from banks' funding costs rising alongside credit spreads, implying banks' net worth is rebuilt very slowly, in contrast to models with a leverage constraint. Furthermore, the increase in fragility due to scarcer net worth gives banks an incentive to demand more liquid assets. This generates a countercyclical liquidity premium.

The monetary and fiscal liabilities of the government are the natural source of liquidity for banks. Banks themselves create liquid assets for other sectors of the economy, but they cannot produce assets that maintain their value when a systemic run occurs.<sup>9</sup> Therefore, the relevant supply of liquid assets is a policy variable. In the model, an increase in the supply of liquidity is expansionary. Extra liquid assets are absorbed on to banks' balance sheets, reducing their fragility. With lower fragility, banks have access to funding on better terms and thus find it optimal to lend more. In other words, supplying more public liquidity crowds in private investment.

In the calibrated model, a shock to liquid assets that reduces the liquidity premium by 15 basis points leads to larger liquid-asset holdings by banks and a reduction in their funding costs by 30 basis points. This effect is larger than our empirical point estimate, but falls within the 99% confidence interval. With cheaper funding, banks expand the supply of credit, reducing credit spreads by 24 basis points. This generates a 2-percent increase in investment on impact, with GDP going up by a quarter of one percent. The supply of liquidity can also be used as a stabilizing policy tool in the face of shocks. If the government responds to banking stress by accommodating banks' increased demand for liquid assets, it dampens the amplification of shocks that otherwise occurs.

The demand for liquid assets gives rise to a fiscal benefit for the government by reducing interest rates on its bonds, analogous to 'seigniorage' in the context of money demand. Interestingly, supplying more liquid assets can have a fiscal cost because it reduces the liquidity premium on government bonds. Thus, a benevolent government may face a trade-off when deciding how much liquidity to supply. In an extension, we study ex-post liquidity policies such as deposit insurance and lender of last resort. These policies also reduce fragility and are expansionary in the same way as the ex-ante supply of liquid assets studied in the main part of the paper. However, providing liquidity ex post rather than ex ante entails a larger fiscal cost for the same economic impact because with the latter, the central bank makes ongoing profits from the liquidity premium.

**Literature review.** An extensive literature builds macroeconomic models around a leverage constraint on banks (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay et al., 2016;

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<sup>9</sup>This is related to the seminal finding in Holmström and Tirole (1998) of a role for public liquidity supply in the presence of aggregate risk.

Mendicino et al., 2020; Di Tella and Kurlat, 2021; Karadi and Nakov, 2021; Van der Ghote, 2021; Fernández-Villaverde et al., 2023).<sup>10</sup> This friction, based on moral hazard on bankers' part, does not naturally give rise to a role for banks' holdings of liquid assets unlike this paper's friction based on coordination failure (both frictions imply an important role for bank net worth). Moreover, models with the moral-hazard friction generate limited shock propagation because adverse shocks to bank net worth push up bank profitability by increasing credit spreads with little change in funding costs. Such models also struggle to match the observed procyclicality of banks' book leverage (Nuño and Thomas, 2017). The coordination friction proposed here makes progress on these two counts. In this paper's model, shock propagation is strong because the positive effect of higher credit spreads on bank profitability after adverse shocks is largely offset by increased funding costs. And we find that leverage is procyclical for standard shocks that affect credit demand, such as productivity shocks.

In this paper, banks demand liquid assets to mitigate the risk of a coordination failure among their creditors. This is a novel source of demand for liquid assets in the macroeconomic literature.<sup>11</sup> The existing literature posits an exogenous risk that bank creditors withdraw their funds. Banks then demand liquid assets as a precaution to limit the amount they must borrow from the central bank at a punitive interest rate (Poole, 1968; Arce et al., 2020; Bianchi and Bigio, 2022) or the amount of assets they must sell at fire-sale prices (Drechsler et al., 2018; D'Avernas and Vandeweyer, 2024; Li, 2025) if hit by an adverse liquidity shock. In our model, the risk of depositor withdrawals is a fully endogenous function of bank fundamentals.

While we model the demand for liquid assets by financial intermediaries, other macroeconomic models focus on other sectors. Households' demand is often modelled as due to liquid assets reducing transaction costs (Krishnamurthy and Vissing-Jorgensen, 2012) or providing insurance against idiosyncratic shocks (Cui and Sterk, 2021; Angeletos et al., 2023). Del Negro et al. (2017) and Kiyotaki and Moore (2019) model demand for liquid assets by firms, which hold them to ensure they can take advantage of investment opportunities that might arise when their ability to borrow is constrained.

Quantitative easing programmes are prominent examples of policies that increased the supply of liquidity. In line with our model, studies evaluating these find they led to lower interest rates (Gagnon et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Chodorow-Reich, 2014; Ray et al., 2024). Acharya and Rajan (2024) stress that some of the reduction in bank fragility from additional liquidity is undone by banks taking on extra leverage. This result conforms to the mechanism in our paper. Diamond et al. (2024) find empirically that liquid-asset holdings increase banks' marginal cost of

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<sup>10</sup>Holmström and Tirole (1997) is an early example of a model in which a leverage constraint on banks is micro-founded with a moral-hazard problem.

<sup>11</sup>A strand of the banking literature formalizes this in static partial-equilibrium models (Rochet and Vives, 2004; Ahnert, 2016).

lending and speculates that the reason for this lies in limited balance-sheet space due to regulation. The effect of regulation is beyond the scope of our paper. On the other hand, the driving force behind our paper’s results, the positive effect of liquid-asset holdings by banks on the demand for their debt, is not considered in [Diamond et al. \(2024\)](#).

Banks’ vulnerability to runs was first formalized in [Diamond and Dybvig \(1983\)](#). That model demonstrates the possibility of runs, but does not speak to their determinants because it features multiple equilibria. A literature has introduced bank runs into standard macroeconomic models and adopted the multiple-equilibria approach to study the macroeconomic effects of runs ([Gertler and Kiyotaki, 2015](#); [Gertler et al., 2016, 2020](#); [Amador and Bianchi, 2024](#)).<sup>12</sup> A limitation of this approach is the need to assume an arbitrary relationship between the probability of runs and fundamentals. Consequently, the role of liquidity in the determination of run risk does not emerge.

Leveraging theoretical results from [Carlsson and van Damme \(1993\)](#), [Goldstein and Pauzner \(2005\)](#) show that a small departure from perfect information produces a unique equilibrium in a bank-run game. This is an attractive feature because the evidence points to a strong relationship between poor bank fundamentals and crises ([Gorton, 1988](#); [Baron et al., 2021](#)). A large literature in banking uses variations of such second-generation bank-run models to study optimal policy ([Vives, 2014](#); [Kashyap et al., 2024](#); [Ikeda, 2024](#); [Leonello et al., 2025](#)). [Ikeda and Matsumoto \(forthcoming\)](#) integrate a second-generation bank-run model into a macroeconomic framework to study bank leverage and the probability of bank runs over the business cycle. We integrate a second-generation bank-run model into a macroeconomic framework to explain banks’ demand for liquid assets and to study the real effects of liquidity policy. Our focus is not on bank runs per se, but on the trade-offs faced by banks trying to avoid runs. We show that these can be tractably summarized in a ‘no-run’ constraint on banks.

**Outline of the paper.** [Section 2](#) contains the empirical analysis. [Section 3](#) and [4](#) respectively lay out the two key parts of the model: the coordination game among depositors, and banks in general equilibrium. [Section 5](#) derives banks’ optimal choices of liquidity demand, deposit creation, and credit supply and their implications for spreads and the liquidity premium. Calibration and the quantitative analysis of macroeconomic shocks are presented in [section 6](#). Finally, [section 7](#) discusses policy implications.

## 2 Empirical analysis

A first step in studying the role played by liquid assets in financial intermediation is to check whether the availability of liquidity matters for banks’ ability to fund lending. [Figure 2](#) shows that the liquidity premium, the price of liquidity, is positively correlated

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<sup>12</sup>A small strand of the banking literature has introduced selected macroeconomic variables into standard bank-run models with the same aim ([Ennis and Keister, 2003](#); [Martin et al., 2014](#); [Porcellacchia, 2020](#); [Mattana and Panetti, 2021](#)).

with banks' funding costs. In this section, we study whether the effect is causal.

**Specification.** We posit the following simple linear empirical model with parameters  $\alpha$  and  $\beta$  linking the bank funding spread  $FS_t$  and the liquidity premium  $LP_t$  at time  $t$ ,

$$FS_t = \alpha + \beta LP_t + \epsilon_t, \quad (1)$$

and allow the error term  $\epsilon_t$  to be correlated with  $L$  lags of a data vector  $\mathbf{y}_t$ , to contain time fixed effects  $\mathbf{d}_t$  and a linear trend.<sup>13</sup> Thus, we can re-write the empirical specification as

$$FS_t = \alpha + \beta LP_t + \sum_{l=1}^L \mathbf{y}_{t-l}^\top \boldsymbol{\zeta}_l + \mathbf{d}_t^\top \boldsymbol{\eta} + \kappa t + \nu_t, \quad (2)$$

where  $\nu_t$  is a stochastic innovation that is serially uncorrelated but is potentially heteroscedastic. Vectors  $\boldsymbol{\zeta}_l$  and  $\boldsymbol{\eta}$  and the scalar  $\kappa$  are parameters.

**Data.** We include in the data vector  $\mathbf{y}_t$  eleven variables at daily frequency with the first observation on 3 January 2006 and the last on 30 June 2023.<sup>14</sup> (1) The funding spread, measured as the difference between 3-month LIBOR and the 3-month general-collateral (GC) repo rate. (2) The liquidity premium, measured as the difference between the 3-month GC repo rate and the 3-month T-bill rate.<sup>15</sup> (3) The quantity of outstanding treasuries, log transformed. (4) The balance on the Treasury General Account, log transformed. (5) The spread between Moody's seasoned Baa corporate bond yield and the 10-year Treasury yield. (6) The S&P 500 stock-market index, log transformed. (7) The S&P 500 financials stock-market index, log transformed. (8) The VIX, log transformed. (9) The 3-month GC repo rate. (10) The 10-year Treasury yield. (11) The trade-weighted exchange rate of the US dollar, log transformed. We set  $L = 80$  to ensure we control for at least one quarter of daily data as lags. Our vector  $\mathbf{d}_t$  includes time dummies for (1) weekdays, (2) days of the month, (3) months, and (4) NBER recessions. The linear time trend does not allow for gaps in the observed dates.<sup>16</sup>

**Identification.** The econometric challenge is to find exogenous variation in the liquidity premium to estimate our parameter of interest  $\beta$ . Because of omitted variables, measurement error, or reverse causality, OLS estimates are unlikely to be consistent. For example, unobserved shocks to uncertainty might be driving both the funding spread and the liquidity premium. Or perhaps the GC repo rate is a noisy measure of the risk-free rate, and measurement error is driving correlation between the measured liquidity premium and funding spread. Another possibility is that shocks to the funding

<sup>13</sup>The data vector also contains the funding spread and liquidity premium.

<sup>14</sup>Before 2006, we do not have daily data on the dollar's trade-weighted exchange rate. The dataset's end date coincides with the final discontinuation date of LIBOR in the US. After merging the series, we are left with 4,157 observations over the period. Data sources are reported in the online appendix.

<sup>15</sup>Our adopted measure of the liquidity premium is standard in the literature (Nagel, 2016; Krishnamurthy and Li, 2023), justified by the much narrower bid-ask spreads for Treasuries than repos. The funding spread is the difference between the rate at which banks can borrow without collateral and the pure risk-free rate absent any liquidity premium as measured by the GC repo rate.

<sup>16</sup>On average, our dataset contains 59 observations per quarter, nearly the universe of business days.

spread are driving the demand for liquidity and thus the liquidity premium.<sup>17</sup>

Our identification strategy is to instrument the liquidity premium with the quantity of outstanding Treasury debt. The quantity of Treasuries is relevant to the liquidity premium as shown in a vast literature studying the convenience yield on these bonds (Krishnamurthy and Li, 2023), and we confirm its relevance in the first-stage regression.

As for the instrument’s validity, Treasury debt is issued a few days after it is auctioned, with a median lag of three days.<sup>18</sup> This institutional feature makes outstanding Treasury debt predetermined at daily frequency, ruling out confounding variables in the innovation  $v_t$  that would make it invalid. It also rules out reverse causality.

Another threat to the instrument’s validity is alternative mechanisms through which the quantity of Treasuries affects the funding spread for a given liquidity premium. We can assuage this concern by noting that an implication of outstanding Treasuries being predetermined at daily frequency is that they are perfectly anticipated. In other words, there is no new information revealed when Treasuries are issued or mature. All information, for instance, regarding fiscal policy, is revealed at latest during auctions. This rules out a direct information effect from the quantity of Treasuries.

Finally, Treasury debt is a highly persistent variable. To rule out a serially correlated omitted variable driving both Treasury debt and the funding spread, it is important that controls included in the regression succeed in removing autocorrelation from the residual. For that, a rich lag structure is needed. Suppose we omitted lags of an element of the true data vector  $y_t$  from the analysis. This would mean the residual contains the omitted lags as well as the stochastic innovation. If in addition to driving the funding spread, the omitted lags were also driving Treasury debt, because for instance they affect fiscal policy, then the instrument would no longer be valid.<sup>19</sup> As described above, we include as controls 80 lags of eleven variables available at daily frequency. As a result, the estimated residuals are not serially correlated.<sup>20</sup>

**Key result.** Table 1 contains the results of the benchmark IV regression. An exogenous one basis-point increase in the liquidity premium increases banks’ funding spread by around 1 basis point.<sup>21</sup> The effect is robustly significant with a p-value of 2.8%.<sup>22</sup>

The instrument is highly relevant as confirmed by the first-stage F statistic of 15. In the first-stage regression, we find that a one-percent increase in Treasuries reduces the

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<sup>17</sup>The results from an OLS regression, reported in the online appendix, are consistent with measurement error in the risk-free rate as a driver of endogeneity.

<sup>18</sup>Data on the time from auction to issuance are reported in the online appendix.

<sup>19</sup>For example, the policymaker could use private information available at the auction date to anticipate the funding spread on the issuance date. If the policymaker exploited this private information to stabilize the funding spread with Treasury issuance then our estimates would be biased downwards.

<sup>20</sup>We report the partial autocorrelation function of the error term in the online appendix.

<sup>21</sup>The effect in the calibrated model is 2 basis points, which is within the 99% confidence interval.

<sup>22</sup>We use heteroscedasticity-consistent standard errors, although a Pagan-Hall general test overwhelmingly fails to reject homoskedasticity of the residuals (the test’s p-value is 100%). With regular standard errors, the p-value is 0.4%.

Table 1: Regression table.

	Funding spread
Liquidity premium	0.99** (0.45)
Lags	Y
Time dummies	Y
Linear trend	Y
R-squared	97%
Observations	4,077
1 <sup>st</sup> -stage F statistic	15

*Note 1:* Outstanding Treasuries are used as an external instrument.

*Note 2:* Heteroscedasticity-consistent standard errors are in parentheses.

*Note 3:* Funding spread = 3M LIBOR – 3M repo rate. Liquidity premium = 3M repo rate – 3M T-bill rate.

liquidity premium by 2.1 basis points (the p-value is 0.3%). The direction is consistent with a movement along a downward-sloping demand curve for Treasuries.

Table 3 in appendix A contains results under alternative specifications. First, we investigate whether there is state dependence in the effect of the liquidity premium on the funding spread. We add as a regressor an interaction term between the liquidity premium and a recession dummy to see to what extent the effect differs according to the state of the economy. As an additional instrument, we use the interaction of Treasuries with the recession dummy. We find that the effect of the liquidity premium on the funding spread in recessions is not significantly different from its effect in expansions. In other specifications, we exclude the time dummies or the lag structure and find that this does not meaningfully affect the results.<sup>23</sup>

In conclusion, we find a causal positive effect of the liquidity premium on the funding spread paid by banks. This is a novel result found using a novel identification strategy. The rest of the paper is dedicated to developing a model to explain this result and investigate its implications for the transmission of macroeconomic shocks and liquidity policy.

### 3 Coordination game

The model is presented in this and the following section, beginning here with the coordination game played by bank depositors. This section solves for the unique equilibrium, which implies a relationship between banks' balance sheets and the interest rates required to induce households to hold deposits.<sup>24</sup> This relationship acts as a constraint on banks in the remainder of the paper. After the game-theoretic analysis

<sup>23</sup>In the online appendix, we also report estimates using different numbers of lags.

<sup>24</sup>Some of the formal results are presented in appendix B.

here, the next section integrates banks into a full general-equilibrium model.

The economy contains a unit continuum of banks (more generally, financial intermediaries) indexed by  $b \in [0, 1]$ . Deposits at bank  $b$  pay interest rate  $j_b$  if held until the next time period, but with an option to withdraw on demand. While referred to as ‘demand deposits’, this bank debt can be interpreted more broadly as short-term unsecured borrowing in money markets that is frequently rolled over.

A coordination game among depositors (bank creditors) is played in each discrete time period, but time subscripts are omitted in this section given the essentially static nature of the game. Just before the coordination game begins, all deposits  $D_b > 0$  at bank  $b$  are equally held by a unit continuum of households indexed by  $h \in [0, 1]$ . Expected payoffs in the next time period are discounted at a common rate  $\rho$  by all households.<sup>25</sup>

**Bank fragility.** Before households decide whether to hold deposits in the coordination game, banks have made portfolio and leverage decisions. Bank  $b$  invests in illiquid and liquid assets  $A_b$  and  $M_b$  respectively, where the notion of liquidity is defined below. Taking as given net worth (equity)  $N_b$ , these choices result in deposit creation up to a level of deposits  $D_b$  consistent with the balance-sheet identity  $A_b + M_b = D_b + N_b$ .

If a positive fraction  $1 - H_b$  of households chooses not to hold deposits  $D_b$  at bank  $b$ , the bank must make a total payment  $(1 - H_b)D_b$  to these households by disposing of some assets. The full value  $M_b$  of the liquid assets acquired earlier can be obtained at this point, but disposal of illiquid assets  $A_b$  during the coordination game only recovers a fraction  $\lambda \in [0, 1]$  of their value at acquisition.<sup>26</sup> If the proceeds of these asset liquidations are insufficient to cover depositor withdrawals, the bank fails. The condition for failure is

$$(1 - H_b)D_b > \lambda A_b + M_b.$$

Rearranging the condition above and using the balance-sheet identity, bank  $b$  does not fail if  $H_b \geq F_b$ , where fragility  $F_b$  is

$$F_b = \frac{(1 - \lambda)A_b - N_b}{A_b + M_b - N_b}. \quad (3)$$

Bank fragility  $F_b$  is the threshold where if the fraction of households holding deposits  $H_b$  is below  $F_b$  then bank  $b$  fails. For positive net worth  $N_b$ , fragility is a number between 0 and  $1 - \lambda$ , and higher net worth lowers fragility. Greater holdings of liquid assets  $M_b$  reduce a bank’s fragility when it is initially positive, while holding more illiquid assets  $A_b$  raises fragility when it is below  $1 - \lambda$  initially. Fragility can be expressed in terms of familiar liquidity and bank capital ratios, respectively

$$m_b = \frac{M_b}{A_b + M_b} \quad \text{and} \quad n_b = \frac{N_b}{A_b + M_b}, \quad \text{as} \quad F_b = \frac{(1 - \lambda)(1 - m_b) - n_b}{1 - n_b}. \quad (4)$$

<sup>25</sup>Since there is a continuum of banks, depositor behaviour can be analysed as if households were risk neutral. The discount rate  $\rho$  is taken as given here, but in the full model,  $\rho$  is an endogenous variable.

<sup>26</sup>A literature studies transaction costs (Grossman and Miller, 1988; Brunnermeier and Pedersen, 2008) and adverse selection (Eisfeldt, 2004) as sources of asset illiquidity.

Hence, a bank's scale plays no role here in determining its fragility.

**Structure of the game.** Households make simultaneous binary choices whether to hold deposits.<sup>27</sup> There is a separate decision for each bank  $b$ . Households' choices are represented by indicator functions  $H_{bh}$ , which equal 1 if household  $h$  holds at bank  $b$  and 0 if  $h$  withdraws. Withdrawing households receive funds in the same time period.<sup>28</sup>

Holding bank deposits may expose households to credit risk because banks can fail. If this happens, those holding deposits only recover the principal after incurring a cost  $\theta$  per unit of deposits. The parameter  $\theta > 0$  represents losses associated with the bankruptcy process. These costs are paid at the beginning of the next time period.<sup>29</sup>

In this environment, banks fail because of 'runs' — too many depositors deciding to withdraw. The share of households who hold bank  $b$ 's deposits is  $H_b = \int_0^1 H_{bh} dh$ , and there is some minimum fraction  $F_b$ , endogenous to the bank's earlier liquidity and leverage choices (equation 3), who must hold for the bank not to fail. The indicator function  $\Phi_b$  for the failure of bank  $b$  depends on comparing  $H_b$  to bank fragility  $F_b$ :

$$\Phi_b(F_b, H_b) = \begin{cases} 0 & \text{if } H_b \geq F_b, \\ 1 & \text{otherwise.} \end{cases} \quad (5)$$

Fragility  $F_b$  is thus the key measure of bank fundamentals in the coordination game.

Conditional on knowing a bank's fragility and the share of households holding its deposits, the net payoff per unit of deposits from holding versus withdrawing is

$$\pi(F_b, H_b) = \frac{(j_b - \rho)(1 - \Phi_b) - \theta \Phi_b}{1 + \rho}, \quad (6)$$

with  $\Phi_b$  given by (5). Households want to hold deposits at bank  $b$  if  $\pi(F_b, H_b) \geq 0$  given their discount rate  $\rho$  and the interest rate  $j_b \geq \rho$  offered by the bank.<sup>30,31</sup> The net payoff  $\pi(F_b, H_b)$  is weakly decreasing in fragility  $F_b$ , representing a deterioration in the bank's fundamentals, and weakly increasing in the fraction  $H_b$  holding deposits, indicating the presence of strategic complementarity in the coordination game. With complete information, there would be multiple Nash equilibria: if  $F_b \in (0, 1]$ , an equilibrium with  $H_{bh} = 1$  where everyone holds, and a 'bank-run' equilibrium with  $H_{bh} = 0$ .

Notice that the illiquidity of assets  $A_b$  is key to the existence of a coordination problem. If the full value of any assets can always be realized, the special case of  $\lambda = 1$ , then banks with non-negative net worth are never fragile. It is also important that banks' portfolio choices are made before households decide whether to hold deposits: once illiquid assets are funded by deposit creation, there is strategic complementarity

<sup>27</sup>To simplify the game, holding deposits is a binary choice, but households would not gain by being able to make partial withdrawals.

<sup>28</sup>For tractability, households must wait until the next time period to deposit funds at another bank.

<sup>29</sup>This timing is not essential; it is chosen for consistency with the full macroeconomic model.

<sup>30</sup>If indifferent, the tie-breaking assumption is that households hold deposits.

<sup>31</sup>Restricting attention to  $j_b \geq \rho$  is without loss of generality because  $j_b < \rho$  makes all households refuse to hold deposits. This is ex-ante suboptimal for a bank.

in depositors' holding decisions. This timing assumption could capture the fact that banks create deposits when they make a loan and then someone in the economy must be willing to hold these deposits if the bank is to avoid having to dispose of assets. More generally, it could be interpreted as a mismatch between the timing of capital investment, which is typically long term, and banks' more short-term funding sources.

**Incomplete information.** In this model, households are subject to an information friction and cannot observe bank  $b$ 's fragility  $F_b$ . As is well known in the literature on global games, such frictions can rule out sunspot-driven bank runs. Each household receives for each bank a private signal  $\hat{F}_{bh} = F_b + \Omega_b + \Sigma_{bh}$  centred around the bank's true fragility with independent systematic and idiosyncratic noise. Systematic noise follows a uniform distribution  $\Omega_b \sim U[-\omega, \omega]$  for some  $\omega > 0$ , and idiosyncratic noise is drawn independently from  $\Sigma_{bh} \sim U[-\sigma, \sigma]$  for some  $\sigma > 0$ .<sup>32</sup> The noise terms represent common and individual-specific errors made in analysing banks' balance sheets.

Formally, in the bank- $b$  coordination game, all of households' prior information is  $\mathcal{I}_b = \{F_b \sim U_{\mathbb{R}}, D_b, j_b\}$ . Household  $h$  updates this prior using signal  $\hat{F}_{bh}$  to form beliefs  $\mathbb{P}_{bh}[\cdot] = \mathbb{P}[\cdot | \hat{F}_{bh}, \mathcal{I}_b]$  and expectations  $\mathbb{E}_{bh}[\cdot] = \mathbb{E}[\cdot | \hat{F}_{bh}, \mathcal{I}_b]$ . The uninformative prior  $F_b \sim U_{\mathbb{R}}$  implies households do not use other sources of information to form beliefs. This assumption can be justified given the subsequent focus on arbitrarily precise signals.<sup>33</sup>

**Equilibrium strategies.** A household  $h$  chooses to hold deposits if and only if there is a non-negative expected net payoff  $\mathbb{E}_{bh}[\pi(F_b, H_b)] \geq 0$  from doing so. We write this condition in terms of the household's belief about bank failure  $\mathbb{E}_{bh}[\Phi_b]$  as

$$j_b - \rho \geq \frac{\mathbb{E}_{bh}[\Phi_b]}{1 - \mathbb{E}_{bh}[\Phi_b]} \theta, \quad (7)$$

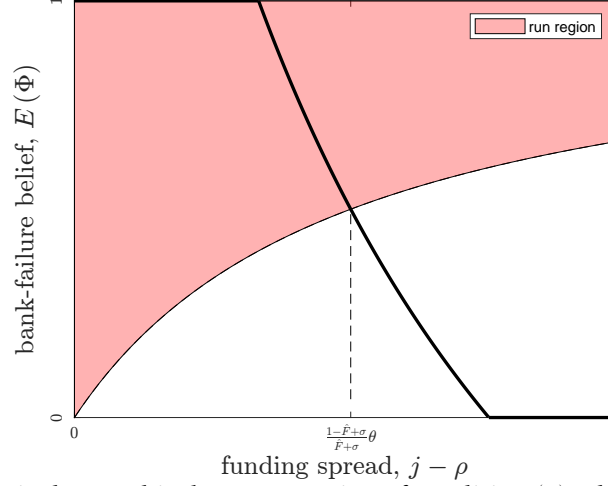
and plot it in [Figure 3](#) as the upward-sloping curve that separates the run region (in red) from the region in which the household holds deposits. According to (7), households demand a premium to compensate for the risk of bank failure. The novelty here is that households' equilibrium beliefs are not pinned down by an exogenous source of risk.

Crucially, the probability of bank failure depends on other households' decisions. Because of a lack of common knowledge, each household is uncertain about the information held by other households and forms beliefs about it. Other households' information is their private signals  $\hat{F}_{bu} = F_b + \Omega_b + \Sigma_{bu}$ . In particular, household  $h$  is interested in forming a belief about the number of other households that, given their information, choose to hold deposits, and whether this number is sufficient to avoid bank failure as described by (5). Conjecturing that other households play a common threshold strategy such that household  $u$  holds deposits if  $\hat{F}_{bu} \leq k_b$ , the number of households holding

<sup>32</sup>The introduction of systematic noise  $\Omega_b$  makes the game's equilibrium outcome stochastic conditional on  $F_b$  and  $j_b$ . As we shall see, this rules out partial runs on the equilibrium path.

<sup>33</sup>A literature studies conditions under which information provided by publicly observed endogenous variables such as policy ([Angeletos et al., 2006](#)) and prices ([Atkeson, 2000](#); [Angeletos and Werning, 2006](#)) restores common knowledge.

Figure 3: Households' decisions to hold deposits.



Note: The run region is the graphical representation of condition (7). The downward-sloping thick line represents equation (8) with  $k_b = F_b^*$  and  $F_b^*$  from equation (9).

conditional on the true fragility and systematic noise is  $G_\Sigma(k_b - F_b - \Omega_b)$ , where  $G_\Sigma(\cdot)$  is the cdf of random variable  $\Sigma_{bu}$ . It follows that household  $h$ 's belief about bank failure is  $\mathbb{P}_{bh}[\Phi_b = 1] = \mathbb{P}_{bh}[G_\Sigma(k_b - F_b - \Omega_b) < F_b] = \mathbb{P}\left[G_\Sigma(k_b + \Sigma_{bh} - \hat{F}_{bh}) < \hat{F}_{bh} - \Sigma_{bh} - \Omega_b\right]$ .

Taking the limit with vanishing systematic noise, that is,  $\omega \rightarrow 0$ , and using the fact that  $\Sigma_{bh} \sim U[-\sigma, \sigma]$ , we can write household  $h$ 's beliefs about bank failure as

$$\mathbb{E}_{bh}[\Phi_b] = \mathbb{P}_{bh}[\Phi_b = 1] = \begin{cases} 0 & \text{if } \hat{F}_{bh} \leq \frac{k_b - 2\sigma^2}{1 + 2\sigma}, \\ \frac{(1 + 2\sigma)\hat{F}_{bh} - k_b + 2\sigma^2}{2\sigma(1 + 2\sigma)} & \text{if } \hat{F}_{bh} \in \left(\frac{k_b - 2\sigma^2}{1 + 2\sigma}, \frac{k_b + 2(1 + \sigma)\sigma}{1 + 2\sigma}\right), \\ 1 & \text{otherwise.} \end{cases} \quad (8)$$

Households believe bank failure is more likely when they receive a higher signal  $\hat{F}_{bh}$  for two reasons. First, it makes a household believe that a bank is more fragile, increasing the number of households that must hold for the bank to survive. Second, a higher signal shifts to the right a household's belief about the distribution of realized signals of all other households. This makes it more likely that fewer households hold deposits. By the same logic, a higher threshold  $k_b$  means each household believes bank failure is less likely because other households are more likely to hold deposits, all else equal.

Using beliefs (8) in combination with condition (7), we can define a threshold for household  $h$ 's signal above which the household does not hold deposits because the offered interest rate  $j_b$  is insufficient compensation for risk. This threshold is a function of the spread of  $j_b$  over the discount rate  $\rho$  and other households' threshold  $k_b$ . In equilibrium, all households play the same threshold strategy  $F_b^*$ . Hence, the unique common threshold is

$$F_b^* = \frac{j_b - \rho}{j_b - \rho + \theta} + \frac{j_b - \rho - \theta}{j_b - \rho + \theta} \sigma. \quad (9)$$

According to equation (9), a higher bank funding spread  $j_b - \rho$  makes households hold deposits for higher realizations of their signals about fragility.<sup>34</sup>

<sup>34</sup>Lemma 1 in appendix B shows formally that the threshold strategy  $F_b^*$  is the game's unique equilib-

A household's equilibrium belief about bank-failure risk is given by equation (8) after substituting in other households' equilibrium threshold strategy  $k_b = F_b^*$  for their deposit-holding behaviour. The bank funding spread  $j_b - \rho$  affects beliefs because it gives other households an incentive to hold deposits. Thus, higher  $j_b - \rho$  reduces the risk of a coordination failure. This relationship between a household's equilibrium belief about bank-failure risk and the funding spread is depicted in Figure 3 as the downward-sloping thick black line. The lowest funding spread at which households hold deposits is where the thick line intersects the boundary of the run region.

With  $\omega \rightarrow 0$ , the funding spread is not a reflection of any extrinsic risk faced by banks. The spread itself is a driver of beliefs about bank failure because it affects others' incentives to hold deposits. Interestingly, seeing a zero funding spread means households choose not to hold deposits for any signal  $\hat{F}_{bh} > -\sigma$  because they think others do not have sufficient incentives to hold. Hence, a bank with strictly positive fragility must pay a strictly positive funding spread to avoid failing.

**No-run condition.** When banks make their choices of leverage, portfolio allocation and deposit rates before the coordination game, their key consideration is the endogenous response of households' deposit-holding decisions. Withdrawals of deposits force banks to dispose of assets and, if large enough, lead to bank failure with the loss of the bank's net worth.

A sufficiently small idiosyncratic component of noise ( $\sigma \rightarrow 0$ ) rules out partial runs on banks because all households make the same decision given that they effectively have the same signal. They either all hold a bank's deposits or no one does. In other words, the share of households holding follows a Bernoulli distribution

$$\mathbb{P}[H_b] = \begin{cases} \kappa_b & \text{if } H_b = 1, \\ 1 - \kappa_b & \text{if } H_b = 0. \end{cases} \quad (10)$$

Using the probability distribution of systematic noise and the equilibrium run threshold (9), we can compute the probability of there being no run

$$\kappa_b = \begin{cases} 1 & \text{if } j_b - \rho \geq \frac{F_b + \omega}{1 - F_b - \omega} \theta, \\ \frac{(j_b - \rho)(1 - F_b + \omega) - (F_b - \omega)\theta}{2\omega(j_b - \rho + \theta)} & \text{if } j_b - \rho \in \left[ \frac{F_b - \omega}{1 - F_b + \omega} \theta, \frac{F_b + \omega}{1 - F_b - \omega} \theta \right), \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

A higher bank funding spread (or lower fragility) increases the probability  $\kappa_b$  all households hold deposits at bank  $b$ . In principle, this creates a trade-off for the bank — it can reduce its funding spread at the cost of a higher risk of a run. At  $j_b - \rho = [(F_b + \omega)/(1 - F_b - \omega)]\theta$ , the steepness  $\partial \kappa_b / \partial j_b$  of this trade-off is captured by the left derivative  $(1 - F_b + \omega)^2 / (2\omega\theta)$  of (11).<sup>35</sup>

From this point on, we assume that systematic noise is small with  $\omega \rightarrow 0$ . In this

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rium. It also gives an implicit expression for the threshold under general distributions of noise.

<sup>35</sup>Proposition 1 in appendix B formally derives equations (10) and (11).

case, the gradient  $\partial \kappa_b / \partial j_b$  of the trade-off becomes vertical: a marginal reduction in the funding spread is so costly in terms of an increase in the run probability that a bank chooses the corner solution with no runs, namely  $\kappa_b = 1$ , which requires

$$j_b - \rho \geq \max \left\{ \frac{F_b}{1 - F_b} \theta, 0 \right\}. \quad (12)$$

This is the no-run condition, which banks choose to comply with because the alternative is a full run and loss of net worth. When we set up a bank's optimization problem in the next section, we introduce the no-run condition (12) as a constraint on the bank.<sup>36</sup>

It is worth studying the no-run condition (12) in greater depth. Substituting in the determinants of bank fragility from equation (3) yields a mapping from the bank's balance sheet to the deposit rate required to avoid a run:

$$j_b - \rho \geq \max \left\{ \frac{(1 - \lambda)A_b - N_b}{\lambda A_b + M_b} \theta, 0 \right\}. \quad (13)$$

Further intuition is gained by substituting the familiar balance-sheet ratios (4) into the no-run condition. The mapping from these to the deposit rate required to avoid runs is

$$j_b - \rho \geq \max \left\{ \frac{(1 - \lambda)(1 - m_b) - n_b}{\lambda + (1 - \lambda)m_b} \theta, 0 \right\}. \quad (14)$$

A graphical representation is provided in Figure 4. The dashed line depicts combinations of the capitalization ratio  $n_b$  and liquidity ratio  $m_b$  that rule out bank failure with a zero spread on deposits. If there is a positive spread on deposits, the region of fundamentals that leads to bank failure, coloured in red, is always below the dashed line. All else equal, a higher interest on deposits makes the failure region smaller. The key implication is that there is a three-way substitutability from a bank's perspective between equity, liquidity, and interest on deposits. For instance, a bank can lever up while keeping its interest-rate expenses in check by boosting its holdings of liquid assets.

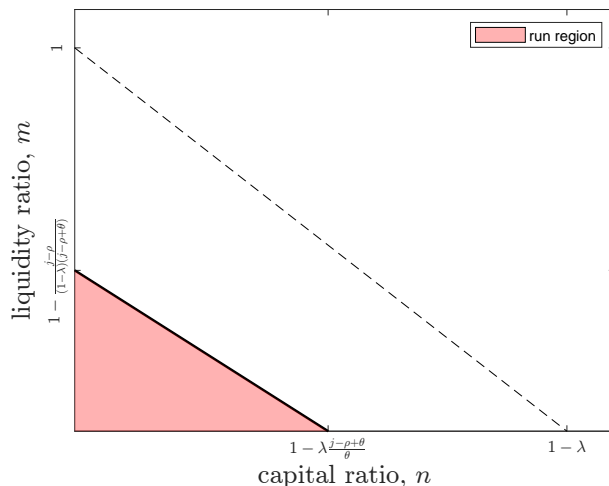
This paper's assumption that both systematic and idiosyncratic noise are small keeps the model as close as possible to the full-information paradigm and parsimonious in terms of parameters. A disadvantage is that we cannot study the effects of a realization of a bank run because banks optimally rule them out on the equilibrium path. Nonetheless, this setting allows us tractably to study those actions that all banks take to avoid bank runs, such as demanding liquid assets, limiting lending, and offering a spread on their debt. These preemptive actions have important macroeconomic consequences, which are the focus of this paper.

Interestingly, the results in this section do not imply that any household's beliefs are incorrect on the equilibrium path. In fact, all households correctly believe that deposits are risk-free as long as idiosyncratic noise is small and banks comply with

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<sup>36</sup>To verify this convenient restriction on a bank's choice set is without loss of generality, once we have laid out the bank's problem in the next section we can check that, (i), the payoff function conditional on  $j_b - \rho$  and  $H_b$  is continuous in  $j_b - \rho$  for any given  $H_b \in (0, 1)$ , and (ii), the payoff given  $H_b = 0$  and  $j_b - \rho \in [0, [F_b/(1 - F_b)]\theta]$  is smaller than the payoff given  $H_b = 1$  and  $j_b - \rho \geq [F_b/(1 - F_b)]\theta$ .

Figure 4: Balance-sheet fundamentals and bank runs.



Note: For a given bank funding spread  $j_b - \rho$ , a run takes place in the red region according to condition (14). The dashed line indicates the boundary of the run region for a zero spread.

the no-run condition.<sup>37</sup> The equilibrium spread on bank deposits is not required as compensation for perceived risk. It is necessary to coordinate households on not running and thereby eliminate the risk of bank failure.

## 4 Macroeconomic model

This section embeds the friction arising from the coordination game played by depositors into a macroeconomic model. To focus on this novel friction, the core of the economy is represented by a real business cycle model (Kydland and Prescott, 1982).

**Timeline.** Each discrete time period  $t = 0, 1, 2, \dots$  is divided into three stages. At the first stage, perfectly competitive markets for goods, labour, physical capital, liquid assets, and illiquid bonds are open. Aggregate shocks and noise in households' signals are realized. Households choose labour supply and non-bank assets, and firms produce final goods and incomes are distributed. The government chooses a supply of liquid assets and sets fiscal policy. During this stage, banks create deposits and set deposit interest rates, select a portfolio of liquid and illiquid assets to hold, and pay dividends. At the second stage, households play the coordination game described in section 3, choosing whether to hold deposits at each bank. At the final stage, households' consumption is determined based on what happened earlier in the period.

**Physical capital as the illiquid asset.** The illiquid assets held by banks are physical capital goods. A surviving bank  $b$  holding illiquid assets  $A_{b,t-1}$  at the end of period  $t - 1$  has a stock of physical capital  $K_{bt} = X_t A_{b,t-1}$  to rent out at price  $p_t$  for use in production of final goods. The random variable  $X_t$ , which has mean 1, represents an exogenous capital-quality shock common to all banks. Capital  $K_{bt}$  depreciates at rate  $\delta$  during each time period. The ex-post return on physical capital between  $t - 1$  and  $t$  is

$$R_t = X_t(1 - \delta + p_t) - 1. \quad (15)$$

<sup>37</sup>Proposition 2 in appendix B derives this result formally.

At the first stage of period  $t$ , final goods can be transformed into new capital through investment  $I_{bt} = A_{bt} - (1 - \delta)K_{bt}$  financed by banks (or existing capital transformed back into final goods if investment is negative). Only goods transformed into capital by this stage can be stored and carried into period  $t + 1$  as physical capital.

Capital is illiquid at the second stage of a time period in the sense that investment is partially irreversible by that point. Only a fraction  $\lambda$  of the physical capital owned by a bank can be immediately converted back into goods usable for consumption without causing the bank to fail. More than this amount can be recovered, but at the cost of bank failure, with the loss of bank equity acting as a form of adjustment cost.<sup>38</sup> In addition, those holding deposits at the point of bank failure must incur a cost  $\theta$  to recover each unit of deposits through the bankruptcy process described in [section 3](#).

**Other frictions.** The macroeconomic relevance of the coordination game among bank depositors depends on three other frictions. First, households cannot directly hold physical capital (banks' illiquid assets), so financial intermediation is necessary for capital accumulation and production. Second, bank debt takes the form of the short-term demand deposits described in [section 3](#), hence there is a mismatch between the liquidity of bank liabilities and assets. Third, banks need positive equity, but face limits on accumulating equity capital, so their assets cannot be financed entirely by equity.

While the model does not speak to why such frictions are present, these are all standard features of the existing macro-banking literature. The first can be justified if holding illiquid assets requires expertise possessed only by bankers, or diversification through the scale at which banks operate. The second can come from some short-term liquidity needs of households that preclude tying up wealth in a long-term investment.

The third is often built into macro-banking models through exogenous exit of banks or bankers. Here, a simpler foundation is a problem of separation of ownership and control of banks. Suppose bank employees are able to divert bank profits to their bonus pools if these funds are not swiftly returned to shareholders. Formally, suppose a constant fraction  $\gamma/(1 + \gamma)$  of pre-dividend net worth is vulnerable to diversion as bonuses  $\Xi_{bt}$ , where  $\gamma$  is a positive parameter. Even if bank  $b$ 's shareholders would otherwise prefer earnings to be retained, they need to pay out at least the funds vulnerable to diversion. This motivates a minimum dividend condition

$$\Pi_{bt} \geq \gamma N_{bt}, \tag{16}$$

where  $\Pi_{bt}$  is bank  $b$ 's dividend and  $N_{bt}$  is net worth after the dividend is distributed.

## 4.1 Production

A continuum of firms  $f \in [0, 1]$  produces homogeneous final goods for consumption or investment. They hire homogeneous labour  $L_{ft}$  at wage  $w_t$  and rent physical

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<sup>38</sup>Banks operate with positive net worth in equilibrium.

capital  $K_{ft}$ . Firms face a constant-returns-to-scale Cobb-Douglas production function

$$Y_{ft} = Z_t K_{ft}^\alpha L_{ft}^{1-\alpha}, \quad (17)$$

where  $Z_t$  is exogenous total factor productivity and  $\alpha$  is the capital elasticity of output.

Goods and factor markets are perfectly competitive and all prices and wages are fully flexible. The price of final goods is normalized to one so that all variables are in real terms. Firms maximize profits  $\Pi_{ft} = Y_{ft} - p_t K_{ft} - w_t L_{ft}$ , which are immediately paid out as dividends. Profit maximization implies capital and labour are hired up to where their marginal products equal the rental price  $p_t$  and the wage  $w_t$  respectively:

$$\alpha Z_t \left( \frac{L_{ft}}{K_{ft}} \right)^{1-\alpha} = p_t, \quad \text{and} \quad (1-\alpha) Z_t \left( \frac{K_{ft}}{L_{ft}} \right)^\alpha = w_t. \quad (18)$$

With constant returns to scale, profits are equal to zero ( $\Pi_{ft} = 0$ ) in equilibrium.

## 4.2 Households

At the beginning of period  $t$ , household  $h \in [0, 1]$  has expected lifetime utility

$$\mathcal{U}_{ht} = \mathbb{E}_{ht} \left[ \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{C_{hs}^{1-\frac{1}{\phi}} - 1}{1 - \frac{1}{\phi}} - \chi \frac{L_{hs}^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \right\} \right], \quad (19)$$

where  $C_{ht}$  is consumption,  $L_{ht}$  is labour supply,  $\beta$  is the subjective discount factor,  $\chi$  is a parameter representing the disutility of labour, and  $\phi$  and  $\psi$  are preference parameters corresponding to the elasticity of intertemporal substitution and Frisch elasticity of labour supply, respectively. All households have the same preferences and begin with equal wealth in period 0. The only heterogeneity is in their signals about bank fragility.

The information set conditioned on in expectation operator  $\mathbb{E}_{ht}[\cdot]$  in (19) contains commonly known aggregate shocks, prices, and macroeconomic variables from date  $t$  and earlier. It also contains household-specific signals, and as analysed in section 3, each household uses its arbitrarily precise signals to form beliefs about banks' fragility.<sup>39</sup>

As explained in section 3, because banks comply with the no-run condition (13), all households have signals that lead them correctly to believe that no runs ever occur.<sup>40</sup> This implies the heterogeneity in households' information sets is irrelevant for choices made at the competitive-markets stage, and hence the household  $h$  subscript can be dropped from the expectation operator. It is replaced by  $\mathbb{E}_t[\cdot]$ , which conditions only on macroeconomic variables known by date  $t$ . Given that bank runs do not happen in equilibrium and no household assigns positive probability to one happening, this section analyses household behaviour abstracting from runs.<sup>41</sup>

<sup>39</sup>They are restricted from using other information to inform their beliefs about fragility. As discussed in section 3, the use of public information could restore common knowledge and thus multiple equilibria.

<sup>40</sup>The formal justification is from Proposition 2 in appendix B together with fragility bounded below  $1 - \lambda$  for positive bank net worth and  $\sigma$  and  $\omega$  being arbitrarily small.

<sup>41</sup>The online appendix describes how actions taken in a run are integrated with the model.

**Competitive-markets stage.** Household  $h$  chooses labour supply  $L_{ht}$  and receives wage income  $w_t L_{ht}$ , and everyone pays a common lump-sum net tax  $T_t$ . Each household receives a non-negative dividend  $\Pi_t$  from owning an equal share of a non-tradable investment fund comprising all banks and non-financial firms,<sup>42</sup> and has an equal claim on the total bonus pool  $\Xi_t$  across all banks. Bonuses are obtained through diversion of bank net worth, though these will be zero because (16) holds in equilibrium.<sup>43</sup>

Household  $h$  may choose to borrow between periods  $t$  and  $t + 1$  an amount  $B_{ht}$  (or if negative, hold savings outside banks) in the form of a risk-free but illiquid bond with interest rate  $\rho_t$ .<sup>44</sup> Any past borrowing  $(1 + \rho_{t-1})B_{h,t-1}$  must be repaid, and a no-Ponzi condition must be respected.<sup>45</sup> Households may also hold a non-negative amount of non-bank liquid assets  $M_{ht}$  paying risk-free interest rate  $i_t$ .

**Budget constraint and utility maximization.** At the start of period  $t$ , households have deposits  $(1 + j_{b,t-1})D_{b,t-1}$  at bank  $b$ , including accrued interest. Bank  $b$ 's net deposit creation  $D_{bt} - (1 + j_{b,t-1})D_{b,t-1}$  funds purchases of physical capital and liquid assets. It is implicit that bank deposits are accepted by firms and households as a means of payment and circulate at the competitive-markets stage.<sup>46</sup> Since non-financial firms are entirely static, paying out all sales revenue immediately as factor payments, all deposits must be in the hands of households once the competitive-markets stage is over.

Conditional on everyone holding deposits and competitive-markets-stage choices, the consumption enjoyed at the final stage of  $t$  is given by the flow budget constraint:<sup>47</sup>

$$C_{ht} = w_t L_{ht} + \Pi_t + \Xi_t - T_t + \int_0^1 \{(1 + j_{b,t-1})D_{b,t-1} - D_{bt}\} db - (1 + \rho_{t-1})B_{h,t-1} + B_{ht} + (1 + i_{t-1})M_{h,t-1} - M_{ht}. \quad (20)$$

Households directly holding physical capital is ruled out by assumption, so capital is excluded from (20). First-order conditions for maximizing utility (19) subject to (20) with respect to  $B_{ht}$  and  $L_{ht}$  determine the optimal choices of consumption and labour supply.<sup>48</sup> With no heterogeneity in wealth or preferences, and no relevant heterogeneity in information sets, consumption  $C_{ht}$  and labour supply  $L_{ht}$  are the same for all  $h \in [0, 1]$ :

$$C_t^{-\frac{1}{\phi}} = \beta(1 + \rho_t)\mathbb{E}_t \left[ C_{t+1}^{-\frac{1}{\phi}} \right], \quad \text{and} \quad \chi L_t^{\frac{1}{\psi}} = w_t C_t^{-\frac{1}{\phi}}. \quad (21)$$

<sup>42</sup>In equilibrium, there are no gains from trading shares in the investment fund among households.

<sup>43</sup>The total bonus pool is  $\Xi_t = \int_0^1 \Xi_{bt} db = \frac{1}{1+\gamma} \int_0^1 \max\{0, \gamma N_{bt} - \Pi_{bt}\} db$ .

<sup>44</sup>Illiquid in that no value from this asset can be realized until the  $t + 1$  competitive-markets stage.

<sup>45</sup>The no-Ponzi condition is  $\lim_{s \rightarrow \infty} \frac{B_{hs}}{(1+\rho_t) \dots (1+\rho_{s-1})} \leq 0$  in all states of the world.

<sup>46</sup>The medium-of-exchange role of deposits is not explicitly modelled here. Households and firms accept deposits in exchange for goods if they believe no bank failures will occur, as is true in equilibrium.

<sup>47</sup>At the final stage of a time period, liquid assets can be traded for consumption goods, and the government can levy additional lump-sum taxes. However, in the absence of bank runs, households do not hold liquid assets at that point and the government has no need to adjust taxes, so these possibilities can be ignored. For a full description of the final stage when runs occur, see the online appendix.

<sup>48</sup>The transversality condition  $\lim_{s \rightarrow \infty} \beta^{s-t} C_{hs}^{-\frac{1}{\phi}} \left( \int_0^1 D_{bs} db - B_{hs} + M_{hs} \right) \leq 0$  must also hold in all states.

There is effectively a representative household from a macroeconomic perspective. Households can also choose to hold liquid assets  $M_{ht} \geq 0$  directly. However, since  $i_t \leq \rho_t$  must hold in equilibrium, utility is maximized by choosing  $M_{ht} = 0$ .<sup>49</sup>

**Coordination game.** Assuming banks treat all households symmetrically when deposits are created, and since all ex-ante identical households behave in the same way at the competitive-markets stage, each household carries the same amount of deposits  $D_{bt}$  at bank  $b$  into the coordination game of section 3. Households' equilibrium strategies in the coordination game maximize expected future consumption payoffs discounted at a common rate. Since there is a continuum of banks  $b \in [0, 1]$ , this is consistent with concave utility in (19) because deposit-holding decisions and bank survival outcomes for any individual bank have only a negligible effect on a household's overall consumption. Household  $h$  therefore acts as risk neutral in respect of deposits at a particular bank, discounting payoffs expected in period  $t + 1$  using discount factor  $\beta \mathbb{E}_{ht} \left[ (C_{h,t+1}/C_{ht})^{-\frac{1}{\phi}} \right]$ . The first-order condition for illiquid bonds (see 21) implies everyone's discount factor equals  $1/(1 + \rho_t)$ . The yield  $\rho_t$  on the illiquid bond is therefore the appropriate discount rate to apply to expected payoffs during the coordination game.

### 4.3 Government

The government issues liabilities  $M_t$  that are liquid assets in the sense that they can be exchanged for consumption goods one-for-one at any stage of period  $t$ .<sup>50</sup> These liabilities are broadly interpretable as government bonds, reserves, or outside money more generally, though the model has a single type of liquid government liability for simplicity. This is an asset that offers a risk-free return  $i_t$  between periods  $t$  and  $t + 1$ .

The government is able to levy lump-sum taxes on households or make transfers. At the competitive-markets stage of period  $t$ , the net lump-sum tax paid by all households is  $T_t$ .<sup>51</sup> The government can also purchase illiquid bonds  $B_t$  (or if negative, issue illiquid bonds). Changes in fiscal and monetary policy are represented through different combinations of  $M_t$ ,  $B_t$ , and  $T_t$ .<sup>52</sup> Consolidating across all branches of government, the flow budget constraint necessary to deliver a risk-free return of  $i_{t-1}$  on  $M_{t-1}$  is

$$T_t = (1 + i_{t-1})M_{t-1} - (1 + \rho_{t-1})B_{t-1} - M_t + B_t. \quad (22)$$

At least one further equation is needed to specify the positive supply of liquidity  $M_t$ .

<sup>49</sup> $M_{ht} = 0$  can be interpreted as households choosing to deposit in banks any outside money obtained from fiscal transfers, and selling any liquid financial assets to banks. Note that if  $i_t > \rho_t$ , there would be an unbounded demand for liquid assets, so an equilibrium must have  $i_t \leq \rho_t$ .

<sup>50</sup>The liquidity of government bonds ultimately derives from the government's ability to adjust the supply of bonds and taxes after the coordination game, as described in the online appendix.

<sup>51</sup>If no runs occur, tax revenue can be collected at the first stage of period  $t$  without loss of generality.

<sup>52</sup>Purchases of illiquid assets  $B_t$  financed by issuing liquid liabilities  $M_t$  can be interpreted as 'quantitative easing', a form of unconventional monetary policy. However, the government never buys physical capital, so it does not directly take on the financial intermediation role performed by banks.

## 4.4 Banks

**Ownership and solvency.** Each bank  $b \in [0, 1]$  and each non-financial firm is owned by a large investment fund, and these funds are themselves owned by households. Investment funds pay out non-negative dividends  $\Pi_t$  to households and they direct the firms they own to act in the interests of households.

This paper focuses on the risk of bank failure caused by illiquidity and runs. To that end alone, we make assumptions so that, in equilibrium, banks do not fail owing to insolvency, which also simplifies the subsequent analysis. Specifically, it is necessary that investment funds recapitalize banks that would otherwise have non-positive equity  $E_{bt}$  at the beginning of time period  $t$ . Injections of capital come from dividends paid out by other banks, and each unit of equity injected costs  $1 + \xi$ , where  $\xi > 0$  is a resource cost of recapitalization.<sup>53</sup> This implies it is optimal for individual banks to act so that the probability of recapitalization is zero. The exogenous stochastic processes  $X_t$  and  $Z_t$  for capital quality and TFP are assumed to have finite support, which means insolvency of the whole banking system — where recapitalization is not feasible — has probability zero in equilibrium. In period 0, banks start with some positive amounts of equity  $\{E_{b0}\}$ .

With no recapitalizations, investment funds aggregate dividends  $\Pi_{bt}$  from banks and any dividends  $\Pi_{ft}$  from non-financial firms and distribute these to households:

$$\Pi_t = \int_0^1 \Pi_{bt} db + \int_0^1 \Pi_{ft} df. \quad (23)$$

**Objectives and choices.** As there is effectively a representative household in equilibrium, the objective function of bank  $b$  is  $\Pi_{bt} + V_{bt}$ , where  $V_{bt}$  is the present value of future dividends (the ex-dividend value of bank  $b$ ) obtained using a stochastic discount factor  $P_{ts}$  given by households' common marginal rate of substitution (see 19) between consumption at date  $t$  and (state-contingent) consumption at date  $s > t$ :

$$V_{bt} = \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} P_{ts} \Pi_{bs} \right], \quad \text{where } P_{ts} = \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-\frac{1}{\phi}}. \quad (24)$$

At each date  $t$ , bank  $b$  chooses a deposit interest rate  $j_{bt}$  and makes a deposit creation decision that results in a stock of deposits  $D_{bt}$ . There is no competitive market for deposits, so a bank can choose both the quantity and the price, but these choices affect whether households decide to hold the bank's deposits during the coordination game. The other choices are the amounts of physical capital  $A_{bt}$  and liquid assets  $M_{bt}$  to hold, and the dividend  $\Pi_{bt}$  to distribute.<sup>54</sup> Each bank is competitive in goods and asset markets and hence takes prices  $(R_t, i_t, \rho_t)$  and the stochastic discount factor  $P_{ts}$  as given.

<sup>53</sup>Equity  $E_{bt} + J_{bt}$  after the injection of capital  $J_{bt}$  costing  $(1 + \xi)J_{bt}$  must be at least  $\underline{E}$  for some  $\underline{E} > 0$ .

<sup>54</sup>Banks do not want to hold illiquid bonds, and they cannot fund themselves by issuing such bonds.

**Constraints.** A bank  $b$  that reaches the beginning of period  $t$  with positive equity has after paying out dividend  $\Pi_{bt}$  the following net worth (bank capital)  $N_{bt} = E_{bt} - \Pi_{bt}$ :

$$N_{bt} = (1 + R_t)A_{b,t-1} + (1 + i_{t-1})M_{b,t-1} - (1 + j_{b,t-1})D_{b,t-1} - \Pi_{bt}, \quad (25)$$

which depends on the bank's assets and liabilities from  $t-1$  and the returns on these. The equation assumes there is no diversion of funds to employee bonuses, which requires the bank to satisfy the minimum-dividend condition (16), but it always finds it optimal to do so.<sup>55</sup> Given net worth  $N_{bt}$  from (25), the balance-sheet identity of bank  $b$  is

$$A_{bt} + M_{bt} = D_{bt} + N_{bt}. \quad (26)$$

With the assumption of small systematic and idiosyncratic noise in households' signals from section 3, banks that do not satisfy the no-run condition (13) face a run with probability one causing them to fail and lose positive net worth  $N_{bt}$ . Instead, by ensuring (13) holds, runs have probability zero and banks make positive profits.<sup>56</sup> Hence, bank  $b$ 's optimal choices maximize the present value of current and future dividends subject to (13) and (16) as constraints, along with (25) and (26).<sup>57</sup> Since net worth  $N_{b,t+1}$  is decreasing in  $j_{bt}$ , the no-run condition (13) must bind when deposits are positive:

$$j_{bt} = \rho_t + \max \left\{ \frac{1}{\lambda + \frac{\lambda N_{bt} + (1-\lambda)M_{bt}}{D_{bt}}} - 1, 0 \right\} \theta. \quad (27)$$

#### 4.5 Aggregation and market clearing

Equilibrium in factor markets requires that non-financial firms rent the physical capital owned by banks and hire the labour supplied by households:

$$\int_0^1 K_{ft} df = \int_0^1 K_{bt} db = K_t, \quad \text{and} \quad \int_0^1 L_{ft} df = \int_0^1 L_{ht} dh = L_t, \quad (28)$$

where the supply of capital  $K_t = X_t A_{t-1}$  depends on banks' aggregate past illiquid assets  $A_{t-1} = \int_0^1 A_{b,t-1} db$  adjusted for the capital quality shock  $X_t$ . Aggregating (17) and (18) across firms implies the aggregate production function  $Y_t = Z_t K_t^\alpha L_t^\alpha$  and labour demand curve  $w_t = (1 - \alpha)Y_t/L_t$ , and the return on capital is  $R_t = X_t(1 - \delta + \alpha Y_t/K_t)$ .

Equilibrium in financial markets requires banks' demand for liquid assets equals the amount supplied by the government, and households' supply of illiquid bonds equals the amount purchased by the government:

$$\int_0^1 M_{bt} db = M_t, \quad \text{and} \quad B_t = \int_0^1 B_{ht} dh, \quad (29)$$

noting that household choose not to hold liquid bonds and banks choose not to hold illiquid bonds. The market for deposits is not perfectly competitive, but households hold the amounts supplied by banks as assumed in the budget constraint (20) since

<sup>55</sup>In general,  $N_{bt} = E_{bt} - \Xi_{bt} - \Pi_{bt}$ . Raising  $\Pi_{bt}$  has no negative effect on  $N_{bt}$  up to where (16) holds.

<sup>56</sup>For completeness, bank  $b$ 's actions in the case of a run are described in the online appendix.

<sup>57</sup>The transversality condition  $\lim_{s \rightarrow \infty} P_{ts} \Pi_{bs} = 0$  in all states of the world is necessary for a maximum. Given the minimum-dividend constraint (16), this implies the restriction  $\lim_{s \rightarrow \infty} P_{ts} N_{bs} = 0$  on net worth.

the no-run condition (27) holds.<sup>58</sup> Combining household, firm, bank, investment fund, and government budget constraints implies market clearing  $C_t + I_t = Y_t$  for final goods, where  $I_t = A_t - (1 - \delta)K_t$  is aggregate investment financed by banks.

## 5 Bank behaviour

This section analyses banks' profit-maximizing choices of asset liquidity, the creation of deposits, the supply of credit to purchase physical capital, and the distribution of dividends subject to the friction developed in the coordination game of section 3. The goal is to explain the empirical findings of section 2 and study the implications for the supply of credit to the economy.

To aid intuition for the main results, banks' full dynamic optimization problem stated in section 4 is solved here as a series of equivalent static problems in liquidity and leverage choices taking as given the path of net worth. Finally, banks' optimal dividend policy is studied to characterize the evolution of net worth.

It is shown later that for bank  $b$  with net worth  $N_{bt}$ , liquid asset demand  $M_{bt}$ , credit supply  $A_{bt}$ , and the total quantity of deposits  $D_{bt}$  created must maximize the expected discounted value of  $N_{b,t+1} + \Pi_{b,t+1}$  using a stochastic discount factor  $\Psi_{t+1}$  common to all banks. This stochastic discount factor differs from that of households and is derived below. The objective function is  $W_{bt} = \mathbb{E}_t[\Psi_{t+1}(N_{b,t+1} + \Pi_{b,t+1})]/\mathbb{E}_t[\Psi_{t+1}]$ , and by using the evolution of net worth (25) and the balance-sheet identity (26),

$$W_{bt} = (1 + r_t)N_{bt} + (r_t - j_{bt})D_{bt} - (r_t - i_t)M_{bt}, \quad (30)$$

where  $r_t = \mathbb{E}_t[\Psi_{t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t+1}]$  denotes the risk-adjusted expected value of  $R_{t+1}$ . In maximizing  $W_{bt}$  with net worth  $N_{bt}$  and  $r_t$ ,  $i_t$ , and  $\rho_t$  given, there are three choice variables  $j_{bt}$ ,  $D_{bt}$ , and  $M_{bt}$  and one binding constraint, the no-run condition (27).

**Demand for liquid assets.** An increase in  $M_{bt}$  given  $D_{bt}$  and  $N_{bt}$  means switching from illiquid to liquid assets while keeping the size of bank  $b$ 's balance sheet unchanged. This has a cost  $r_t - i_t$ , as seen from (30), reflecting the difference in (risk-adjusted) expected returns between the two assets, referred to as the credit spread since  $r_t$  is the return on supplying credit for capital accumulation. The benefit of more liquidity is the fall in bank fragility,  $F_{bt} = 1 - \lambda - ((1 - \lambda)M_{bt} + \lambda N_{bt})/D_{bt}$  from (3) and (26), which lowers the bank's funding cost. The binding no-run constraint (27) gives the deposit interest rate  $j_{bt}$  as a function of  $M_{bt}$  and  $D_{bt}$ , and (30) implies the marginal benefit is equal to  $-\partial j_{bt}/\partial M_{bt}$  multiplied by deposits  $D_{bt}$ . If fragility is positive, the marginal benefit is

$$-D_{bt} \frac{\partial j_{bt}}{\partial M_{bt}} = (1 - \lambda)\theta \left( \lambda + \frac{(1 - \lambda)M_{bt} + \lambda N_{bt}}{D_{bt}} \right)^{-2} = \frac{(1 - \lambda)\theta}{(1 - F_{bt})^2}, \quad (31)$$

<sup>58</sup>The transversality condition on households' asset holdings together with the no-Ponzi condition on household borrowing, the non-negativity of deposits, and  $M_{ht} = 0$  imply the transversality condition  $\lim_{s \rightarrow \infty} \mathbb{E}_t \left[ P_{ts} \int_0^1 D_{bs} db \right] = 0$  on deposits using the formula for the stochastic discount factor in (24).

but if fragility is already negative then the marginal benefit is zero.

If  $r_t - i_t > (1 - \lambda)\theta$ , in which case the bank's demand for liquid assets leaves it with positive fragility, the first-order condition maximizing (30) with respect to  $M_{bt}$  is  $r_t - i_t = -D_{bt}\partial j_{bt}/\partial M_{bt}$ .<sup>59</sup> If  $r_t - i_t = 0$ , the bank demands enough  $M_{bt}$  to ensure fragility is negative, while if  $0 < r_t - i_t \leq (1 - \lambda)\theta$ , the bank targets zero fragility exactly when choosing liquid assets. Using (3), (26), and (31), bank  $b$ 's demand for liquid assets is

$$M_{bt} \begin{cases} = \frac{1}{1-\lambda} \left( \sqrt{\frac{(1-\lambda)\theta}{r_t - i_t}} - \lambda \right) D_{bt} - \frac{\lambda}{1-\lambda} N_{bt} & \text{if } r_t - i_t > (1 - \lambda)\theta \\ = D_{bt} - \frac{\lambda}{1-\lambda} N_{bt} & \text{if } 0 < r_t - i_t \leq (1 - \lambda)\theta \\ \geq D_{bt} - \frac{\lambda}{1-\lambda} N_{bt} & \text{if } r_t - i_t = 0 \end{cases} \quad (32)$$

Demand for liquidity is decreasing in the cost  $r_t - i_t$  of holding liquid assets, increasing in deposits  $D_{bt}$  because more leverage increases fragility, and decreasing in net worth  $N_{bt}$  because bank capital is a substitute for liquidity in reducing fragility.<sup>60</sup>

Since all banks face the same cost  $r_t - i_t$  of holding liquid assets and the marginal benefit depends only on an individual bank's fragility  $F_{bt}$ , banks trade liquid assets up to the point where fragility is equalized across them.<sup>61</sup> This is analogous to the demand for reserves in [Poole \(1968\)](#) arising from payments risk. With  $F_{bt} = F_t$  for all  $b$ , systemic bank fragility  $F_t$  is derived from (3) by aggregating the balance-sheet identities (26):

$$F_t = 1 - \lambda - \frac{(1 - \lambda)M_t + \lambda N_t}{D_t}, \quad (33)$$

where  $N_t$ ,  $M_t$ , and  $D_t$  are the aggregate amounts of equity, liquid assets, and deposits in the banking system. A consequence of  $F_{bt} = F_t$  is that all banks face the same minimum funding cost  $j_t = j_{bt}$  consistent with a binding no-run constraint. Using (27) and (33), this deposit rate satisfies  $j_t - \rho_t + \theta = \theta/(1 - F_t)$  for non-negative fragility, and combining with (31) shows the marginal benefit of liquid assets common to all banks after trading is positively related to banks' funding cost  $j_t$ . By equating this to the cost of liquidity:

$$r_t - i_t = \frac{(1 - \lambda)\theta}{(1 - F_t)^2} = \frac{(1 - \lambda)}{\theta} (j_t - \rho_t + \theta)^2 \quad \text{if } F_t > 0. \quad (34)$$

**Deposit creation.** An increase in deposits  $D_{bt}$  given  $M_{bt}$  and  $N_{bt}$  means greater leverage, with bank  $b$  increasing the size of its balance sheet (26). Since banks trade liquid assets so as to equalize fragility for any given  $D_{bt}$ , the objective function (30) can be

<sup>59</sup>The second-order condition is satisfied because (31) shows  $-D_{bt}\partial j_{bt}/\partial M_{bt}$  is decreasing in  $M_{bt}$ .

<sup>60</sup>The non-negativity constraint  $M_{bt} \geq 0$  means corner solutions must be checked. However, a corner solution for some banks but not others can be ruled out because a binding non-negativity constraint reduces the maximum attainable  $W_{bt}$ , but it will be seen that banks are indifferent about the size of  $D_{bt}$ , and the non-negativity constraint is slack for sufficiently large  $D_{bt}$  (see 32). Furthermore, a positive aggregate supply of liquidity  $M_t$  means that there cannot be a corner equilibrium for all banks.

<sup>61</sup>The marginal benefit depends only on fragility because the reduction in fragility is inversely proportional to deposits, but the lower funding cost is multiplied by the size of the deposit base.

written in terms of the common funding cost  $j_t$  and systemic fragility  $F_t$  using (33):<sup>62</sup>

$$W_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1 - \lambda}\right) N_{bt} + \left(\frac{r_t - i_t}{1 - \lambda} F_t + i_t - j_t\right) D_{bt}, \quad (35)$$

This is linear in deposits, so if the coefficient of  $D_{bt}$  is positive, there is no limit to banks' desire to create deposits, while if negative, no deposit creation occurs. Hence, an equilibrium with a positive but finite supply of deposits requires that the coefficient on  $D_{bt}$  is zero.<sup>63</sup> If fragility is zero, this means that  $i_t = j_t = \rho_t$ . With  $F_t > 0$ , a rearrangement of the coefficient of  $D_{bt}$  shows that it is zero when  $(r_t - \lambda i_t)/(1 - \lambda) = j_t + (r_t - i_t)(1 - F_t)/(1 - \lambda)$ . Using the implication (34) of banks' demand for liquidity, this is equivalent to:

$$\frac{r_t - \lambda i_t}{1 - \lambda} = j_t + (j_t - \rho_t + \theta), \quad \text{where } j_t - \rho_t + \theta = \frac{\theta}{1 - F_t}. \quad (36)$$

The terms on the right-hand side are respectively the direct funding cost of the additional deposit and the cost of holding the additional liquid assets to avoid raising fragility, which is also positively related to banks' funding costs. Taking as given  $r_t$ ,  $i_t$ , and  $\rho_t$ , the aggregate supply of deposits  $D_t$  adjusts until the deposit rate  $j_t$  satisfies equation (36), with higher  $D_t$  increasing systemic fragility (33) and hence raising  $j_t$ .<sup>64</sup> In what follows, attention is restricted to cases where deposits  $D_t$  are strictly positive.

**The liquidity premium and aggregate demand for liquidity.** Equation (34) shows the difference between the returns  $r_t$  and  $i_t$  on banks' illiquid and liquid assets is positively related to banks' funding spread of  $j_t$  over the risk-free rate  $\rho_t$ . Intuitively, the funding spread reflects banks' fragility, and thus their demand for liquid assets. Together with  $j_t - i_t = (r_t - i_t)F_t/(1 - \lambda)$  from the zero coefficient on  $D_{bt}$  in (35) for the supply of deposits, (34) implies  $j_t - i_t = (j_t - \rho_t)(j_t - \rho_t + \theta)/\theta$ . Thus, a high funding spread lowers the yield  $i_t$  on liquid assets relative to other interest rates, including  $\rho_t$  on illiquid bonds. Simplifying the equation shows banks' funding spread is a geometric average of the liquidity premium  $\rho_t - i_t$  and depositors' loss given default parameter  $\theta$ :

$$j_t - \rho_t = \sqrt{\theta} \sqrt{\rho_t - i_t}. \quad (37)$$

The definitions of the funding spread and liquidity premium here are analogous to the empirical spreads between LIBOR, the GC repo, and T-bills in that  $j_t$  is unsecured, and while both  $\rho_t$  and  $i_t$  are risk-free yields, government bonds have the advantage of liquidity. Equation (37) implies a positive relationship between the liquidity premium and the funding spread that explains theoretically the empirical findings in section 2.<sup>65</sup>

<sup>62</sup>Intuitively, if  $D_{bt}$  increases by one unit, but fragility remains unchanged at  $F_t$  after adjusting liquid assets, (3) implies the composition of the increase in total assets is  $A_{bt}$  rising by  $F_t/(1 - \lambda)$  and  $M_{bt}$  rising by  $1 - F_t/(1 - \lambda)$ . This delivers an additional payoff of  $(r_t - i_t)F_t/(1 - \lambda) + i_t$  for the bank at the cost of paying extra interest  $j_t$ , but with no further effect on overall funding costs, hence the coefficient of  $D_{bt}$  in (35).

<sup>63</sup>Deposits are zero in equilibrium only if  $r_t \leq \rho_t$ . This is because fragility must be negative if  $D_t = 0$ , hence  $r_t = i_t$  and  $j_t = \rho_t$ , so the coefficient of  $D_{bt}$  is  $r_t - \rho_t$ .

<sup>64</sup>Note that the exact distribution of deposits  $D_{bt}$  across banks is not uniquely determined, only the aggregate amount of deposits  $D_t$  consistent with (36). With reference to (32), this ensures that the earlier non-negativity constraint  $M_{bt} \geq 0$  on liquid assets can be ignored without loss of generality.

<sup>65</sup>Notice that the empirical model (1) corresponds to a linearized version of equation (37).

Combining equations (34) and (37) shows that there is also a positive relationship between the liquidity premium and the credit spread  $r_t - i_t$ :

$$r_t - i_t \begin{cases} = 4(1 - \lambda) \left( \frac{1}{2} \sqrt{\theta} + \frac{1}{2} \sqrt{\rho_t - i_t} \right)^2 & \text{if } F_t > 0 \\ \in [0, (1 - \lambda)\theta] & \text{if } F_t = 0, \\ = 0 & \text{if } F_t < 0 \end{cases} \quad (38)$$

which is a multiple  $4(1 - \lambda)$  of a generalized mean of  $\rho_t - i_t$  and  $\theta$  when fragility is positive. The term  $1 - \lambda$  captures the difference in liquidity of banks' assets  $A_t$  and  $M_t$ .

Conditional on the amount of credit  $A_t$  supplied by banks, the liquidity premium and other spreads are jointly determined by banks' aggregate demand for liquid assets and the supply  $M_t$  of these assets resulting from government policies. In the case  $F_t > 0$ , by aggregating equation (32) and using (38), banks' total demand for liquid assets is<sup>66</sup>

$$M_t = \frac{\sqrt{\theta}((1 - \lambda)A_t - N_t)}{\sqrt{\rho_t - i_t}} - \lambda A_t, \quad (39)$$

noting that  $(1 - \lambda)A_t > N_t$  if and only if fragility is positive. The demand for liquidity is decreasing in the liquidity premium  $\rho_t - i_t$ , with a horizontal asymptote as  $\rho_t - i_t$  approaches zero. The demand curve shifts to the right as bank holdings of illiquid assets  $A_t$  increase, and to the left if net worth  $N_t$  is higher. When  $(1 - \lambda)A_t \leq N_t$ , which means fragility is non-positive, the liquidity premium must be zero, but (32) is consistent with any holdings of liquid assets, so the demand curve for  $M_t$  is horizontal at  $\rho_t - i_t = 0$ .

The supply curve is determined by government policy. The supply of  $M_t$  may be inelastic, or alternatively have some response to interest-rate spreads such as  $\rho_t - i_t$  and  $r_t - i_t$ . Since spreads move together with the liquidity premium (37 and 38), the supply curve is represented as a relationship between  $M_t$  and the liquidity premium  $\rho_t - i_t$ :

$$M_t = M_t^* e^{\eta(\rho_t - i_t)}, \quad (40)$$

where  $\eta$  is the semi-elasticity of  $M_t$  with respect to the liquidity premium, and  $M_t^*$  is an exogenous variable capturing any other shifts in the supply of liquid assets.

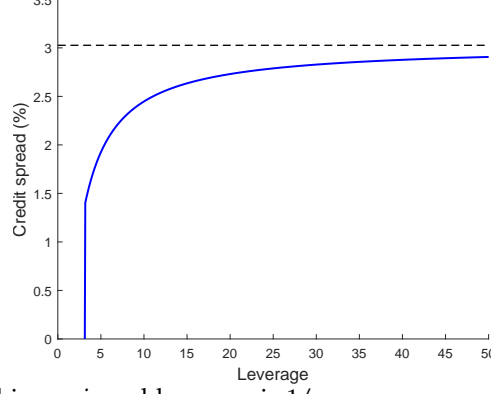
**The credit supply curve.** While individual banks can choose holdings of liquid assets, in equilibrium, the banking system must hold the liquidity  $M_t$  supplied by the government. Hence, banks' supply of deposits can be seen as determining the supply of credit  $A_t$ , taking as given  $M_t$  and aggregate net worth  $N_t$ . From equation (32):

$$A_t = \begin{cases} \frac{(\sqrt{r_t - i_t} - \sqrt{(1 - \lambda)\theta})M_t + \sqrt{(1 - \lambda)\theta}N_t}{\sqrt{(1 - \lambda)\theta} - \lambda\sqrt{r_t - i_t}} & \text{if } r_t - i_t > (1 - \lambda)\theta \\ \frac{N_t}{1 - \lambda} & \text{if } 0 < r_t - i_t \leq (1 - \lambda)\theta \end{cases}. \quad (41)$$

This supply curve for credit implies that a higher credit spread  $r_t - i_t$  induces banks to increase their leverage  $1/n_t = (A_t + M_t)/N_t$ , and thus increase their fragility according to (33). The credit supply curve is depicted in Figure 5 for an inelastic supply of liquid

<sup>66</sup>This is derived by noting that (32) holds for aggregates because the coefficients are the same for all banks, and then substituting  $D_t = A_t + M_t - N_t$  and  $\sqrt{\frac{r_t - i_t}{(1 - \lambda)\theta}} - 1 = \sqrt{\frac{\rho_t - i_t}{\theta}}$  from the formula in (38).

Figure 5: The credit supply curve.



Note 1: The credit spread is  $r_t - i_t$  and leverage is  $1/n_t$ .

Note 2: Annualized calibrated parameter values from Table 2 are used.

Note 3: The dashed line is the spread at which credit supply is unlimited.

assets  $M_t$ . For low levels of the credit spread, the supply of credit is inelastic at the point where banks are not fragile and they pay the risk-free rate on deposits. Above a given credit spread, banks have an incentive to lever up and become fragile. In this region, the supply of credit is elastic. Increases in the supply of liquid assets expand credit supply in the fragile region, but they are irrelevant when banks are not fragile.

**Dividend policy and net worth.** Up to this point, net worth  $N_{bt}$  has been taken as given. The remaining decision to analyse is the distribution of dividends  $\Pi_{bt}$ .

Since the coefficient on deposits  $D_{bt}$  in (35) is zero (or if not, deposits are zero), the static objective function  $W_{bt}$  from (30) is linear in net worth only:

$$W_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1 - \lambda}\right) N_{bt}. \quad (42)$$

The evolution of net worth depends on the return on equity and banks' dividends. Defining  $Q_{b,t+1}$  as the ex-post return on bank  $b$ 's book equity between  $t$  and  $t + 1$ :

$$Q_{b,t+1} = \frac{\Pi_{b,t+1} + (N_{b,t+1} - N_{bt})}{N_{bt}}, \quad \text{thus } Q_{b,t+1} = R_{t+1} \frac{A_{bt}}{N_{bt}} + i_t \frac{M_{bt}}{N_{bt}} - j_t \frac{D_{bt}}{N_{bt}}, \quad (43)$$

where the latter uses (25). The risk-adjusted expected return on bank  $b$ 's book equity is  $q_{bt} = \mathbb{E}_t[\Psi_{t+1} Q_{b,t+1}] / \mathbb{E}_t[\Psi_{t+1}]$  evaluated using the stochastic discount factor  $\Psi_{t+1}$  introduced earlier. The definition  $W_{bt} = \mathbb{E}_t[\Psi_{t+1} (N_{b,t+1} + \Pi_{b,t+1})] / \mathbb{E}_t[\Psi_{t+1}]$  of the static objective function and (43) imply that  $q_{bt} = (W_{bt} - N_{bt}) / N_{bt}$ , so bank behaviour analysed up to this point can be thought of as maximizing the risk-adjusted expected return on book equity conditional on initial net worth  $N_{bt}$ . Equation (42) shows this maximized expected return is the same for all banks,  $q_t = q_{bt}$ , and is given by

$$q_t = \frac{r_t - \lambda i_t}{1 - \lambda}. \quad (44)$$

Bank  $b$ 's actual objective function in choosing the path of dividends and other balance-sheet variables is the present value of current and future dividends discounted using the representative household's stochastic discount factor  $P_{ts}$ . This means maximizing  $\Pi_{bt} + V_{bt}$ , where  $V_{bt}$  is the present value of future dividends from (24).<sup>67</sup>

<sup>67</sup>The detailed solution is derived in the online appendix, with the key results presented here.

Optimization by banks implies the present value of future dividends  $V_{bt}$  is proportional to net worth  $N_{bt}$ , with the market-to-book ratio  $v_t = V_{bt}/N_{bt}$  being common to all banks. The optimal choices of bank  $b$ 's portfolio of assets and deposit creation also maximize the static objective function  $W_{bt}$  in (30) defined for some stochastic discount factor  $\Psi_{t+1}$ , hence the earlier analysis of bank behaviour correctly characterizes the solution to the full dynamic optimization problem with the appropriate stochastic discount factor  $\Psi_{t+1}$ . This modifies the representative-household stochastic discount factor  $P_{t,t+1}$  from (24) depending on the future market-to-book ratio  $v_{t+1}$  of banks. An expression for  $\Psi_{t+1}$  and the expectational difference equation satisfied by  $v_t$  are

$$\Psi_{t+1} = \left(1 + \frac{v_{t+1} - 1}{1 + \gamma}\right) P_{t,t+1}, \quad \text{and} \quad v_t = \left(\frac{1 + \frac{r_t - \lambda_t}{1 - \lambda}}{1 + \rho_t}\right) \left(1 + \frac{\mathbb{E}_t[P_{t,t+1}(v_{t+1} - 1)]}{(1 + \gamma)\mathbb{E}_t[P_{t,t+1}]}\right). \quad (45)$$

One result is that the market-to-book ratio  $v_t$  is never lower than 1, and  $v_t > 1$  implies the minimum dividend constraint (16) is binding. Using the ex-post return on book equity  $Q_{b,t+1}$  from (43), the evolution of net worth if the constraint is binding at  $t + 1$  is

$$N_{b,t+1} = \left(\frac{1 + Q_{b,t+1}}{1 + \gamma}\right) N_{bt}. \quad (46)$$

The minimum dividend constraint (16) binds when the parameter  $\gamma$  is sufficiently large. There is a range of  $\gamma$  values for which net worth converges to a positive steady state in the absence of shocks, and it is assumed parameters are in this range in the remainder of the paper.<sup>68</sup> Starting from that steady state, the minimum dividend constraint will always be binding for some bounds on the size of aggregate shocks.

## 6 Quantitative analysis

This section quantifies the importance of the coordination friction in the transmission of shocks. The model dynamics have a tractable solution by log linearization.<sup>69</sup>

### 6.1 Calibration

The banking sector of the economy is described by the three parameters,  $\lambda$ ,  $\theta$ , and  $\gamma$ . These are calibrated using information on the average liquidity premium, credit spread, and return on bank equity. The parameter  $\beta$  is calibrated using average interest rates. Parameters are chosen to match the model's implications for targeted variables in a non-stochastic steady state to the averages observed. The policy-determined supply of liquid assets consistent with the steady-state liquidity premium is inferred from the average capitalization ratio of banks. Finally, the other macroeconomic parameters  $\alpha$ ,  $\delta$ ,  $\phi$ , and  $\psi$  are set to conventional values following the literature.

The model is calibrated to U.S. economy using data from 1991 up to the 2007–8 financial crisis. Data availability for banking variables determines the start of the

<sup>68</sup>This range of values of  $\gamma$  is analysed when solving for the model's steady state in the online appendix.

<sup>69</sup>The non-stochastic steady state and the log linearization are derived in the online appendix.

sample in 1991Q3, and stopping in 2008Q4 accounts for the substantially different provision of liquidity after 2008 resulting from the many policy responses to the crisis.

The liquidity premium is defined with reference to the 3-month Treasury bill as the most liquid asset. The average T-bill yield over the period 1991Q3–2008Q4 is 3.7% in nominal terms. In the model, all interest rates are in real terms, so the average 2.2% rate of inflation over the same period according to the personal consumption expenditure (PCE) price index is subtracted, leaving a real yield of 1.5%. The macroeconomic model is formulated in discrete time, and it is natural to align the length of one period with the 3-month maturity of the T-bill. The steady-state quarterly real interest rate on the liquid asset is  $i$ , so  $i = 1.5\%/4$ , where a variable without a time subscript denotes its non-stochastic steady-state value. The liquidity premium  $\rho - i$  as measured by the 3-month GC repo rate minus the T-bill yield is 28 basis points on average, thus  $\rho = i + 0.28\%/4$ .

The credit spread  $r - i$  for illiquid bank assets is proxied by the yield on Moody's seasoned Baa-rated corporate bonds over 10-year Treasuries, which is 2.2% annual, hence  $r = i + 2.2\%/4$ . In the model, the steady-state real return on bank equity coincides with the dividend-net worth ratio. Hence, the return on bank equity  $q$  is measured by the average ratio of cash dividends to equity for commercial banks covered by the Federal Deposit Insurance Corporation (FDIC), which is 8.4% annual, giving  $q = 8.4\%/4$ .<sup>70</sup>

Since  $r = (1 - \lambda)q + \lambda i$  from (44), the parameter  $\lambda$  measuring the liquidity of bank assets is calibrated as  $\lambda = (q - r)/(q - i)$ . As the formula shows, a low value of  $\lambda$  arises if  $r$  is large relative to  $i$  because the illiquidity of assets makes it challenging for banks to supply credit without increasing fragility. The calibration targets imply  $\lambda = 0.681$ .

The parameter  $\theta$  measuring the costs of bank failure for depositors is calibrated with information on  $\rho$ ,  $i$ , and  $q$ . Using equations (38) and (44),  $q - i = (\sqrt{\theta} + \sqrt{\rho - i})^2$ , so  $\theta$  is set as  $\theta = (\sqrt{q - i} - \sqrt{\rho - i})^2$ . High values of  $\theta$  arise when the return on bank equity  $q$  is far above the risk-free interest rate  $\rho$  because a more severe credit friction increases spreads. The value resulting from the calibration targets is  $\theta = 4.4\%/4$ .

In a steady state where the return on bank equity exceeds the risk-free rate, the return on equity  $q$  is equal to the minimum fraction  $\gamma$  of equity distributed as dividends. This immediately implies  $\gamma = 8.4\%/4$ . In steady state, households' Euler equation from (21) implies the discount factor  $\beta$  satisfies  $\beta = 1/(1 + \rho)$ . With  $\rho = 1.78\%/4$ , the implied discount factor is  $\beta = 0.996$ . The calibrated parameters are given in Table 2.

The observed liquidity premium as the price of liquidity effectively pins down, along with the other spreads, the quantity of liquidity supplied in the steady state by the government. Using equation (33) for bank fragility, the steady-state liquidity ratio is

$$m = 1 - \left( \frac{q - i}{r - i} \right) \left( n + (1 - n) \sqrt{\frac{\rho - i}{q - i}} \right),$$

<sup>70</sup>The nominal return on bank equity for FDIC banks is 11.6%, implying an annual real return of 9.4%, which is close to the dividend-equity ratio.

Table 2: Calibrated parameters of the model.

Description	Notation	Value
Bank-asset liquidity relative to T-bills	$\lambda$	0.681
Loss given bank default	$\theta$	4.4%/4
Minimum dividend distribution	$\gamma$	8.4%/4
Subjective discount factor	$\beta$	0.996
Elasticity of intertemporal substitution	$\phi$	1
Frisch elasticity of labour supply	$\psi$	3
Capital elasticity of output	$\alpha$	1/3
Depreciation	$\delta$	7.5%/4
Steady-state liquidity ratio	$m$	0.148

where  $n$  is the steady-state capital ratio. Using data on total equity capital and total assets from the FDIC, the average bank capital ratio is 8.8%. This implies  $m = 0.148$ .

The macroeconomic parameters are set following the literature. The elasticity of intertemporal substitution  $\phi$  is 1 and the Frisch elasticity of labour supply  $\psi$  is 3. The capital elasticity of output  $\alpha$  is set to 1/3 to match the capital share of national income. The depreciation parameter  $\delta$  is chosen to give a 7.5% annualized depreciation rate.

## 6.2 Results

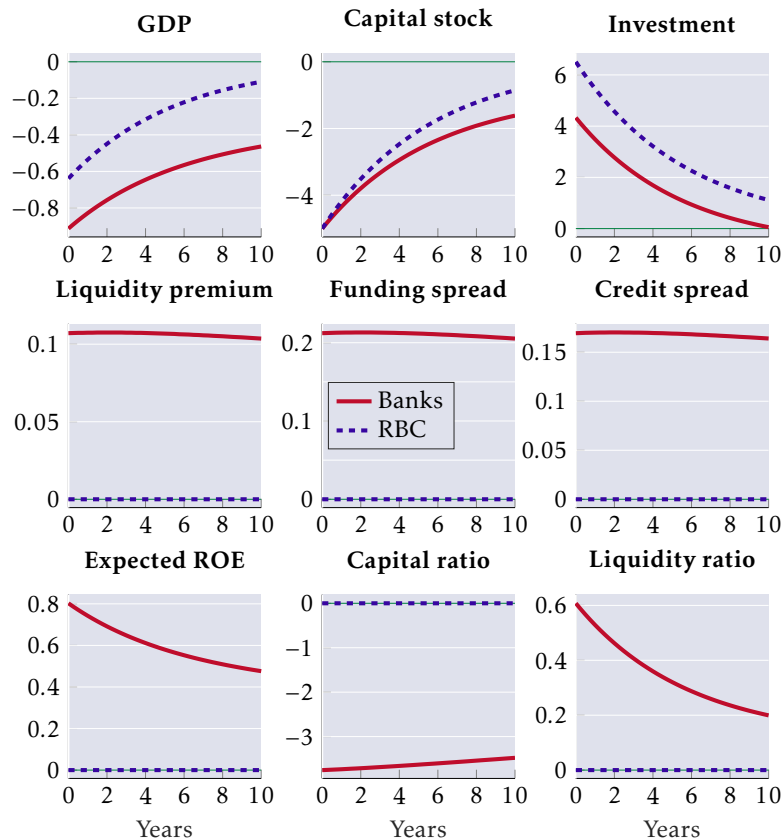
**Capital destruction shocks.** We simulate the model to show the effects of a one-off capital destruction shock, that is, an unexpected negative shock to  $X_t$ . Formally,  $X_t = 1 + v_t$ , where  $v_t$  is a zero-mean i.i.d. shock with support on  $[-\zeta, \zeta]$  for some  $\zeta > 0$ . To begin with, we assume government policy is completely passive and the supply of liquid assets is not adjusted, that is,  $\eta = 0$  and  $M_t^* = M$  in (40). Impulse response functions of key macroeconomic and banking variables for 10 years after a 5% shock ( $v_t = -0.05$ ) are shown as the red solid lines in Figure 6 labelled ‘Banks’. Variables such as interest rates, spreads, and ratios are percentage-point deviations from steady state (annualized for interest rates and spreads), with 1 meaning 1 percentage point. All other variables are percentage deviations from steady state, with 1 denoting 1%.

As a point of comparison, consider an RBC model with the same macroeconomic features but no banking sector. In that model, households directly hold physical capital, but to make the steady states comparable, there is an exogenous but time-invariant spread between the risk-adjusted return on capital  $\hat{r}_t$  and the risk-free rate  $\rho_t$ :

$$\hat{r}_t = \rho_t + (r - \rho), \quad \text{where } \hat{r}_t = \frac{\mathbb{E}_t[P_{t,t+1}R_{t+1}]}{\mathbb{E}_t[P_{t,t+1}]}, \quad (47)$$

and  $r - \rho$  is the steady-state spread between  $r_t$  and  $\rho_t$  in the model with banks. This equation replaces banks’ credit supply function (41), but all other equations for household and firm behaviour in the RBC model are also found in the banking model.

Figure 6: Impulse response functions following a capital destruction shock.



The responses of macroeconomic variables in the RBC model to the capital destruction shock are shown in Figure 6 as the blue dashed lines labelled ‘RBC’. Note that spreads are either constant or absent from the RBC model, as are variables related to banks. The shock directly reduces the capital stock by 5%, which brings down GDP. The RBC model effectively captures the frictionless response to the shock, hence investment rises so that the marginal product of capital is tied to the risk-free interest rate.

In the model with banks, the loss of some of the assets held by banks reduces their equity and capital ratio (leverage is countercyclical for this shock). Banks’ fragility rises and this causes them to demand more liquid assets, pushing up the liquidity premium  $\rho_t - i_t$  by 11 basis points. Greater fragility means banks must offer a higher interest rate on deposits to avoid runs, with the funding spread  $j_t - \rho_t$  rising by 21 basis points. The increase in funding costs reduces banks’ supply of credit, causing the credit spread  $r_i - i_t$  to rise by 17 basis points. This results in less investment and a slower recovery of the capital stock compared to the RBC model. Consequently, GDP is lower and returns to its steady state at a slower rate. The amplification of the shock to GDP is quantitatively important, being around one third on impact and larger at longer horizons.

**Endogenous persistence.** One notable feature of the quantitative results is the slow return of variables to their steady states after a transitory shock. This endogenous persistence comes from the behaviour of net worth, which approaches its steady state

only gradually. Using the expected return on equity  $q_t = \mathbb{E}_t[Q_{b,t+1}]$  common to all banks, (46) implies the expected path of aggregate net worth is  $\mathbb{E}_t N_{t+1} = (1 + q_t)N_t/(1 + \gamma)$ . The low rate of convergence to the steady state is accounted for by  $q_t$  rising by a relatively small amount when banks' equity  $N_t$  falls after a shock.

Aggregating equation (43) for banks' returns on equity and balance sheets (26):

$$q_t = \rho_t + (r_t - \rho_t) \frac{(1 - m_t)}{n_t} - (j_t - \rho_t) \frac{(1 - n_t)}{n_t} - (\rho_t - i_t) \frac{m_t}{n_t}, \quad (48)$$

which shows that the difference between the expected return on bank equity  $q_t$  and the risk-free rate  $\rho_t$  can be decomposed into terms that depend on the spread between  $r_t$  and  $\rho_t$ , banks' funding spread  $j_t - \rho_t$ , and the liquidity premium  $\rho_t - i_t$ . These spreads are scaled by terms that depend on the aggregate bank capitalization and liquidity ratios  $n_t = N_t/(A_t + M_t)$  and  $m_t = M_t/(A_t + M_t)$ .

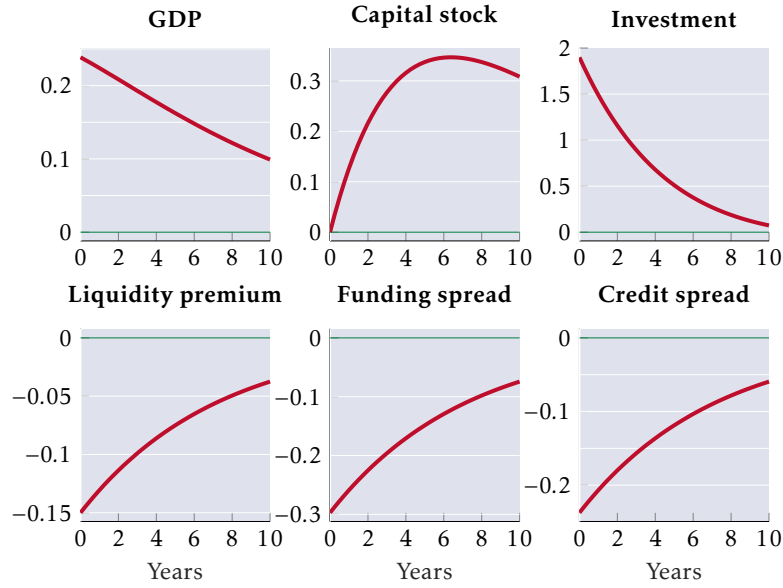
In an environment without bank fragility where bank funding costs  $j_t$  are equal to the yield on government bonds  $i_t$  and the risk-free rate  $\rho_t$ , banks would generate an expected return on equity of  $q_t = i_t + ((1 - m_t)/n_t)(r_t - i_t)$ . The credit spread  $r_t - i_t$  is multiplied by a factor  $(1 - m_t)/n_t$ , reflecting the magnifying effect of the leverage ratio  $1/n_t$  on the expected return  $r_t$  from a fraction  $1 - m_t$  of bank assets. With a lower supply of credit after a shock, the credit spread  $r_t - i_t$  rises, which leads to a large increase in the return on equity given high bank leverage. Models abstracting from the bank fragility that gives rise to funding spreads and the liquidity premium therefore imply that equity can be rebuilt rapidly after a negative shock, limiting endogenous persistence.

When banks are fragile as in the model here, lower equity also means an increase in funding costs  $j_t$ , and a higher funding spread  $j_t - \rho_t$  reduces banks' return on equity according to (48). The funding spread is multiplied by  $(1 - n_t)/n_t$ , which is large given bank leverage. Moreover, since the demand for liquid assets increases, the liquidity premium  $\rho_t - i_t$  rises, which is multiplied by  $m_t/n_t$  in (48), also reducing the return on equity through a lower overall return on banks' portfolio of assets. Taken together, these novel effects significantly reduce the rise in the expected return on bank equity after a capital destruction shock, resulting in a very high degree of endogenous persistence.

**Liquidity shocks.** The no-run constraint (13) implies that the quantity of liquid assets held by banks matters for their fragility in addition to their net worth. Since government policies affect the aggregate supply of liquid assets, this opens up a bank lending channel through which fiscal or monetary policy can affect the economy.

We simulate the effects of an increase in liquidity by considering an exogenous shift in policy such that there is an unexpected 15 basis points decline in the liquidity premium  $l_t = \rho_t - i_t$  with a half-life of 5 years. Formally, the quantity of liquid assets  $M_t$  is adjusted to target a liquidity premium equal to  $l_t^*$ , with the deviation of  $l_t^*$  from the steady-state liquidity premium  $l$  following the AR(1) process  $l_t^* - l = a(l_{t-1}^* - l) + v_t$ , where

Figure 7: Impulse response functions following an expansion of liquid assets.



$v_t$  is a zero-mean i.i.d. shock with support  $[-\zeta, \zeta]$ .<sup>71</sup> The autoregressive parameter  $a$  is set so that the persistence of  $l_t^* - l$  matches the 5-year half-life. The impulse response functions of macroeconomic variables and spreads are shown in Figure 7.

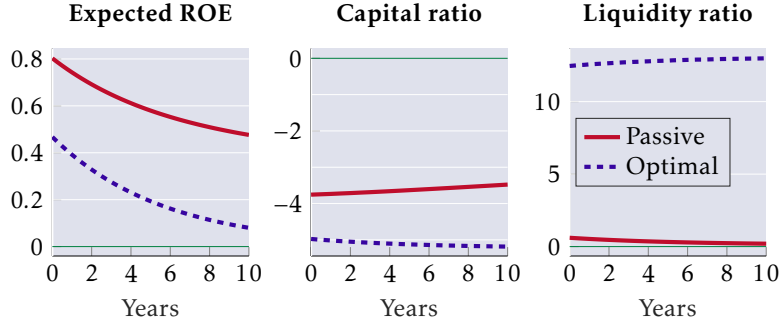
The reduction in fragility due to the expansion of liquidity causes banks' funding costs to fall and leads them to lever up. The funding spread falls by 30 basis points, and the credit spread by 24 basis points. There is a rise in investment, which boosts GDP. Observe that leverage is procyclical for liquidity shocks due to government policy.

**Stabilizing the liquidity premium.** We can also study the supply of liquidity as a systematic response to shocks. Optimal policy is discussed formally in section 7, but it is natural to think of an elastic response of liquid assets to accommodate changes in demand as desirable. Suppose the government supplies sufficient  $M_t$  to keep the liquidity premium  $\rho_t - i_t$  constant at its initial steady state  $l$  after the capital destruction shock considered earlier. Since other spreads are linked to the liquidity premium (see 37 and 38), this policy also completely stabilizes the bank funding spread and credit spread by offsetting the effect of the shock on bank fragility. Consequently, to a first-order approximation, the response to the shock is now the same as in the benchmark RBC model (see 47), and the difference between a passive and elastic supply of liquidity can be seen by comparing the 'Banks' and 'RBC' impulse responses in Figure 6.

To stabilize spreads, the supply of liquid assets must increase significantly and persistently, with banks' liquidity ratio rising by 12 percentage points (Figure 8). The persistence is necessary because absent changes to spreads, bank equity does not recover.

<sup>71</sup>This is implemented with a perfectly elastic supply of liquidity ( $\eta \rightarrow \infty$  in 40). The supply of  $M_t$  is what is consistent with the aggregate demand for liquidity (39) at the target liquidity premium  $\rho_t - i_t = l_t^*$ .

Figure 8: Stabilizing the liquidity premium after a capital destruction shock.



## 7 Liquidity policy

This section studies the supply of liquidity from a normative perspective.

**First best.** As a benchmark, a social planner assigns equal consumption  $C_t$  and labour supply  $L_t$  to each household to maximize expected lifetime utility (19) subject only to resource constraints, namely the aggregate production function  $Y_t = Z_t K_t^\alpha L_t^{1-\alpha}$ , the constraint  $C_t + I_t = Y_t$  on utilization of the economy's output, and the capital accumulation equation  $K_{t+1} = X_{t+1}(I_t + (1 - \delta)K_t)$ . The first-order conditions of this problem are  $(1 - \alpha)Y_t/L_t = \chi C_t^{-\frac{1}{\phi}} L_t^{\frac{1}{\psi}}$  and  $\beta \mathbb{E}_t \left[ (C_{t+1}/C_t)^{-\frac{1}{\phi}} X_{t+1} (\alpha Y_{t+1}/K_{t+1} + 1 - \delta) \right] = 1$ . Except for the final one, all of these constraints and first-order conditions are equilibrium conditions of the market economy with banks (see section 4 and equations 18 and 21).

To judge whether the planner's first-order condition with respect to capital holds in the economy with banks, consider the expected return  $\hat{r}_t$  on physical capital, risk-adjusted using the representative household's stochastic discount factor  $P_{t,t+1}$ , as defined for the benchmark RBC economy in (47). The first best is attained in the market economy with banks if and only if  $\hat{r}_t = \rho_t$ , where  $\rho_t$  is the yield on an illiquid but risk-free bond.

**The liquidity premium as a capital wedge.** From the credit spread formula in (38) with positive fragility,  $r_t - \rho_t = (1 - \lambda) (\sqrt{\theta} + \sqrt{l_t})^2 - l_t$ , where  $l_t = \rho_t - i_t$  is the liquidity premium and  $r_t = \mathbb{E}_t[\Psi_{t,t+1} R_{t+1}] / \mathbb{E}_t[\Psi_{t,t+1}]$  is the risk-adjusted expected return on physical capital using banks' stochastic discount factor  $\Psi_{t+1}$  from (45). The spread between  $r_t - \rho_t$  is increasing in the liquidity premium  $l_t$ . Taking a first-order approximation around a steady state with no aggregate risk,  $\hat{r}_t \approx r_t$ , and hence the wedge between  $\hat{r}_t$  and  $\rho_t$  is approximately equal to  $r_t - \rho_t$ , which is increasing in the liquidity premium. Therefore, in an economy with banks, a large liquidity premium acts as a wedge between the expected return on capital and households' discount rate. Government policies that increase the supply of liquidity and reduce the liquidity premium thus act to move the economy closer to the first best by reducing the size of the capital wedge.

**Liquidity policy cannot implement the first best.** While a lower liquidity premium improves efficiency, liquidity supply policy cannot implement a first-best allocation of

resources. Even if the liquidity premium were zero, the wedge  $r_t - \rho_t$  remains positive (assuming bank net worth is scarce, so fragility is not negative, see 38). Furthermore, the shape of the aggregate demand curve for liquidity (39) shows the liquidity premium cannot be reduced to zero with any large but finite supply of liquid assets. Therefore, the capital wedge cannot be entirely eliminated by the government supplying liquidity.

**Stabilizing spreads.** While the capital wedge cannot be closed with liquidity policy, the government can stabilize the size of the wedge with an elastic supply of liquid assets. By adjusting  $M_t$  to target the steady-state positive liquidity premium  $l$ , banks' funding spread, the credit spread, and the capital wedge also remain at their steady-state levels (see 37 and 38). This generally requires permanent changes in the supply of liquidity following temporary shocks because the expected return on bank equity (44) is also held at its steady-state level, so bank equity does not revert to its mean after a shock.

**Substitutability between liquidity and bank capital.** The policy described above is based on there being some substitutability between liquid assets and bank capital in managing bank fragility. The credit supply curve (41) can be expressed equivalently as follows using (39) for a given supply of liquid assets  $M_t$ :

$$A_t = \frac{\sqrt{\rho_t - i_t} M_t + (1 - \lambda) \sqrt{\theta} N_t}{(1 - \lambda) \sqrt{\theta} - \lambda \sqrt{\rho_t - i_t}}.$$

If a shock changes net worth  $N_t$ , the adjustment of liquidity supply  $M_t$  needed to maintain the same supply of credit at the same liquidity premium and other spreads is

$$\left. \frac{\partial M_t}{\partial N_t} \right|_{\rho_t - i_t, A_t} = - \frac{(1 - \lambda) \sqrt{\theta}}{\sqrt{\rho_t - i_t}}. \quad (49)$$

The required size of the liquidity response to falls in net worth is decreasing in  $\rho_t - i_t$ . When liquidity is abundant and the premium is low, a larger response of liquid assets is needed to stabilize spreads, reflecting a form of diminishing returns to liquidity.

**Fiscal implications of liquidity policy.** Iterating forwards the government's flow budget constraint (22), and using  $\lim_{s \rightarrow \infty} \mathbb{E}_t[P_{ts}(M_s - B_s)] = 0$  implied by the transversality and no-Ponzi conditions, yields a present-value government budget constraint:

$$\sum_{s=t}^{\infty} \mathbb{E}_t[P_{ts} T_s] = (1 + i_{t-1}) M_{t-1} - (1 + \rho_{t-1}) B_{t-1} - \sum_{s=t}^{\infty} \mathbb{E}_t[P_{ts} \tau_s], \quad \text{where } \tau_t = \frac{(\rho_t - i_t) M_t}{1 + \rho_t}.$$

This states that the present value of current and future taxes  $T_t$  must equal initial government liabilities  $(1 + i_{t-1}) M_{t-1}$  net of initial government assets  $(1 + \rho_{t-1}) B_{t-1}$ , minus the present-value of the fiscal gain  $\tau_t$  from the government's ability to supply liquid assets.<sup>72</sup> This fiscal gain derives from a positive liquidity premium  $\rho_t - i_t$ , which means the government is able to borrow at a lower rate than issuers of illiquid bonds.

Policies that reduce the liquidity premium  $\rho_t - i_t$ , which move the economy closer

<sup>72</sup>Ricardian equivalence holds in respect of tax policy  $T_t$  and the government's supply or purchase of illiquid bonds  $B_t$ . Only policies that change  $M_t$  have an impact on the economy, and whatever combination of  $T_t$  and  $B_t$  is implemented to satisfy the government budget constraint does not matter.

to first best, can have a fiscal cost. If the present value of  $\tau_t$  falls, the present value of taxes  $T_t$  must increase to satisfy the government's budget constraint.<sup>73</sup>

**The liquidity Laffer curve.** Using the aggregate demand curve for liquid assets (39):

$$(\rho_t - i_t)M_t = \sqrt{\theta} \sqrt{\rho_t - i_t} ((1 - \lambda)A_t - N_t) - \lambda(\rho_t - i_t)A_t.$$

This shows that as  $M_t$  rises and moves  $\rho_t - i_t$  towards zero, the fiscal gain  $\tau_t$  to the government approaches zero. In other words, the elasticity of liquid asset demand with respect to the liquidity premium is less than one, so a large enough supply of liquidity eventually pushes the government's total fiscal gain towards zero. Higher government borrowing costs are thus a drawback of large expansions in the supply of liquid assets.

Note however that more liquidity does not necessarily mean lower fiscal gains. As  $M_t$  approaches zero, the liquidity premium  $\rho_t - i_t$  rises, but only by a finite amount, which implies  $\tau_t$  also approaches zero. Hence, there is a 'Laffer curve' for the fiscal gains deriving from the government's supply of liquid assets.

**Ex-ante versus ex-post liquidity supply.** The liquidity policies studied so far are essentially changes in the supply of liquid assets held ex ante by banks before run risk materializes. But ex-post provision of liquidity can also be analysed in this framework.

Suppose the government or central bank offers a discount window facility whereby banks can exchange illiquid assets for liquid assets. Assume the central bank applies a 'haircut'  $\lambda_t^*$  at date  $t$ , where  $\lambda_t^* > \lambda$ .<sup>74</sup> All the analysis of section 3 and 5 goes through as before with the parameter  $\lambda$  replaced by the policy variable  $\lambda_t^*$ .<sup>75</sup>

Alternatively, suppose the government sets up a system of deposit insurance whereby those holding deposits at failing banks now suffer a loss  $\theta_t^*$  smaller than  $\theta$ .<sup>76</sup> This is analysed by replacing parameter  $\theta$  with the policy variable  $\theta_t^*$  in all equations.

With these new policy instruments, the aggregate supply of credit (41) is now

$$A_t = \frac{\sqrt{\rho_t - i_t}M_t + (1 - \lambda_t^*)\sqrt{\theta_t^*}N_t}{(1 - \lambda_t^*)\sqrt{\theta_t^*} - \lambda_t^*\sqrt{\rho_t - i_t}},$$

which is increasing in  $\lambda_t^*$  and decreasing in  $\theta_t^*$ . Hence, a rise in credit supply, or equivalently, a lower credit spread and liquidity premium, can also be achieved with more ex-post liquidity, that is, higher  $\lambda_t^*$  or lower  $\theta_t^*$ , as well as more ex-ante liquidity  $M_t$ .

Since the ex-post liquidity facilities are not used on the equilibrium path, there is no direct change to the government's budget constraint, and  $\tau_t = (\rho_t - i_t)M_t/(1 + \rho_t)$  remains the fiscal gain derived from supplying liquid assets. But as all policies affect

<sup>73</sup>While this model has no distortions arising from (lump-sum) taxes  $T_t$ , more realistic representations of the tax system entail deadweight losses from increases in the government's fiscal needs.

<sup>74</sup>Implicitly, this facility is backed by the government's tax-raising powers at the final stage of period  $t$ . See the online appendix on bank runs in the macroeconomic model for a formal treatment.

<sup>75</sup>Since banks still want to avoid runs, the facility is not used on the equilibrium path, but its presence affects the outcome of the coordination game.

<sup>76</sup>Again, backed by the government's tax-raising powers. The cost  $\theta_t^*$  is paid at the beginning of  $t + 1$ . On the equilibrium path, deposit insurance is not used, but its presence affects the coordination game.

the equilibrium liquidity premium  $\rho_t - i_t$ , ex-post liquidity is not a free lunch for the government, even though the facilities are not used. Their availability reduces banks' desire to hold liquid assets ex ante, lowering the liquidity premium. By effectively raising the government's borrowing costs, ex-post liquidity policies have a fiscal cost.

Moreover, if the same reduction in  $\rho_t - i_t$  is achieved through  $\lambda_t^*$  or  $\theta_t^*$  without an increase in  $M_t$ , the reduction in  $\tau_t$  is larger than when higher  $M_t$  is used to lower  $\rho_t - i_t$ . Thus, ex-post liquidity provision to reduce  $\rho_t - i_t$  is more expensive to the government than the same change brought about through an expansion of liquidity ex ante.

## 8 Conclusion

This paper has developed a novel financial friction based on coordination failure in the market for bank deposits. The friction implies that fragile banks borrow on worse terms. Liquid-asset holdings and net worth are substitutable factors that reduce banks' fragility. Hence, when net worth is scarce, banks demand more liquid assets. Introducing this friction into a canonical macroeconomic model, we showed the model matches the positive correlation of the liquidity premium with indicators of financial stress. Current macroeconomic models with financial frictions do not speak to this fact. Moreover, the model points to a role for policy to stabilize the economy by adjusting the supply of liquid assets. Empirically, we tested a key prediction of the model: a high liquidity premium leads to high funding costs for banks. Exploiting exogenous variation in the liquidity premium at daily frequency due to predetermined changes in the supply of Treasuries, we found a robustly significant positive effect. The calibrated model's prediction is within the 99% confidence interval of the empirical estimate.

The paper provides a quantitative framework to understand and evaluate policies that change the supply of liquid assets in the economy. A case in point is quantitative easing, as enacted in response to the financial disruptions of the global financial crisis. The current generation of macroeconomic models largely appraises this policy as a credit policy: QE is effective because the central bank makes loans that banks cannot make on account of a binding leverage constraint. In this paper's framework, the real effects of QE stem from the liability-side of the central-bank balance sheet regardless of its asset holdings. Abundant liquid reserves on banks' balance sheets make creditors willing to lend to banks on more favourable terms. The two effects are not exclusive: there is scope for combining the moral-hazard and coordination frictions for a broader account of central-bank balance-sheet policies. More generally, the interaction of liquidity supply with other policy levers warrants further investigation. For this, the introduction of additional frictions such as distortionary taxes or nominal rigidities is necessary.

# A Figures

Figure 9: Pandemic and tightening cycle.

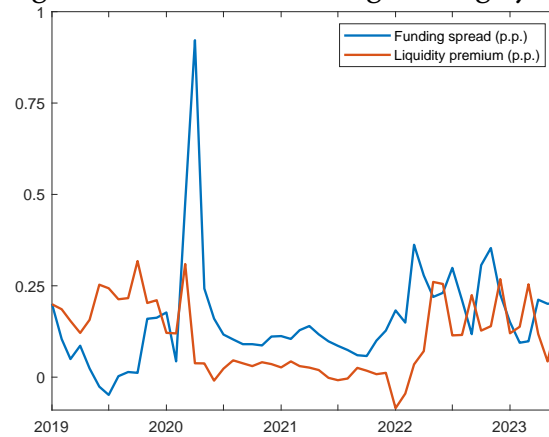
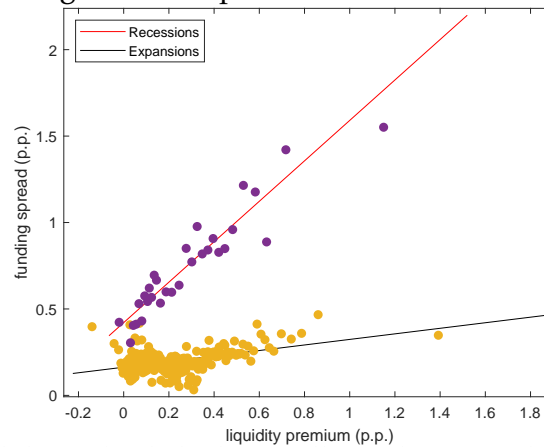


Figure 10: Expansions vs recessions.



Note: Scatterplot of binned data. The daily data is allocated to quantile-based bins according to the liquidity premium. There are 270 bins for expansions and 30 bins for recessions.

Table 3: IV with alternative specifications

Funding spread	IV	IV	IV	IV	IV
Liquidity premium	1.4 (1.0)	1.0** (0.48)	0.31*** (0.04)	1.28*** (0.06)	0.99** (0.45)
Liquidity premium × Recession	-0.54 (1.0)				
Lags	Y	Y	N	N	Y
Time dummies	Y	N	Y	N	Y
Linear trend	Y	Y	Y	Y	Y
R-squared	96%	96%	57%	17%	97%
Observations	4077	4077	4157	4157	4077
1 <sup>st</sup> -stage F statistic	3.9	13	1560	1823	15

Note 1: IV estimation uses outstanding treasuries as external instrument.

Note 2: In first column, outstanding treasuries × recession used as additional external instrument.

Note 3: Heteroskedasticity-consistent standard errors in parentheses.

Note 4: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

## B Proofs

Because the global game introduced in [section 3](#) is standard, we do not present the formal steps of the argument in the main text. They are in this appendix.

**Lemma 1.** *In equilibrium, there is a unique common threshold  $F_b^*$  such that household  $h$  holds bank  $b$ 's deposits if and only if  $\hat{F}_{bh} \leq F_b^*$ . For  $\omega \rightarrow 0$ , this threshold is given by*

$$F_b^* = \frac{j_b - \rho}{j_b - \rho + \theta} + \frac{j_b - \rho - \theta}{j_b - \rho + \theta} \sigma. \quad (9)$$

*Proof.* A strategy in the coordination game is a correspondence that maps a household's signal  $\hat{F}_{bh}$  into the deposit-holding decision  $H_{bh}$ .

Consider other households playing the same threshold strategy such that they hold a bank's deposits with  $H_{bh} = 1$  if they receive signal  $\hat{F}_{bh} \leq k_b$  and do not hold the deposits otherwise. Given household  $h$ 's (improper) uniform prior and signal  $\hat{F}_{bh}$  about bank fragility, its expected net payoff of holding deposits can be written as

$$\tilde{\pi}^*(\hat{F}_{bh}, k_b) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\pi}(\hat{F}_{bh}, k_b, \Sigma_{bh}, \Omega_b) g_{\Sigma}(\Sigma_{bh}) g_{\Omega}(\Omega_b) d\Sigma_{bh} d\Omega_b, \quad (50)$$

where  $\tilde{\pi}(\hat{F}_{bh}, k_b, \Sigma_{bh}, \Omega_b)$  is the net payoff from holding deposits given the unknown noise and  $g_x(\cdot)$  is a general pdf for a random variable  $x$ .

Conditional on the noise, the share of households holding the deposits of bank  $b$  is  $H_b = G_{\Sigma}(k_b - \hat{F}_{bh} + \Sigma_{bh})$ . Together with the definition of bank failure (5), this result implies that we can write a condition for  $\Sigma_{bh}$  such that the bank fails if and only if  $\Sigma_{bh} < \underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b)$  with  $\underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b)$  solving the following implicit equation:

$$\underline{\Sigma} = \hat{F}_{bh} - \Omega_b - G_{\Sigma}(k_b - \hat{F}_{bh} + \underline{\Sigma}). \quad (51)$$

Importantly, the solution to this equation is unique because the left-hand side is continuous and increasing in  $\underline{\Sigma}$ , while the right-hand side is continuous and decreasing in  $\underline{\Sigma}$ .

We can now re-write the expected net payoff from holding deposits as

$$\tilde{\pi}^*(\hat{F}_{bh}, k_b) = \mathbb{E}\left\{G_{\Sigma}\left[\underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b)\right]\right\}(-\theta) + \left(1 - \mathbb{E}\left\{G_{\Sigma}\left[\underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b)\right]\right\}\right)(j_b - \rho), \quad (52)$$

where the expectation is taken with respect to the unknown systematic noise  $\Omega_b$ .

Now, we start iteratively to delete dominated strategies. First, we consider a strategy of holding deposits if and only if  $\hat{F}_{bh} \leq \underline{F}$  with  $\underline{F}$  large enough. This strategy implies holding when  $\hat{F}_{bh} \rightarrow +\infty$  and in this case we have that  $\tilde{\pi}^*(\hat{F}_{bh}, k_b) = -\theta$  for any  $k_b$ . Hence, this is a dominated strategy and no household will play it.

We can extend this logic by studying  $\hat{\pi}^*(z) = \tilde{\pi}^*(z, z)$ . If other households play threshold strategy  $z$ , does household  $h$  have an incentive to hold if it receives  $\hat{F}_{bh} = z$ ? Without loss of generality, we consider threshold strategies for other households because, due to strategic complementarities, they are the non-dominated strategy that makes household  $h$ 's expected net payoff from holding deposits highest. In other words, given that holding deposits for  $\hat{F}_{bh} > z$  is dominated if other households play threshold strategy  $z$  and household  $h$  is better off not holding for  $\hat{F}_{bh} = z$ , then holding deposits for  $\hat{F}_{bh} = z$  is also

dominated. Function  $\hat{\pi}^*$  is monotonically decreasing and crosses zero once at  $z^*$  with

$$j_b - \rho = \frac{\mathbb{E}\{G_\Sigma[\underline{\Sigma}(\Omega_b, z^*, z^*)]\}}{1 - \mathbb{E}\{G_\Sigma[\underline{\Sigma}(\Omega_b, z^*, z^*)]\}}\theta. \quad (53)$$

With this, we can delete as dominated strategies such that a household holds deposits with  $\hat{F}_{bh} > z^*$ . We can apply this analysis in reverse to delete as dominated all strategies that set  $H_{bh} = 0$  for  $\hat{F}_{bh} < z^*$ . Hence, we are left with a unique equilibrium strategy.

Furthermore, using the fact that idiosyncratic noise  $\Sigma_{bh}$  follows a uniform distribution  $U[-\sigma, \sigma]$ , we obtain

$$\underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b) = \begin{cases} \hat{F}_{bh} - \Omega_b & \text{if } \Omega_b > k_b + \sigma, \\ \frac{(1+2\sigma)\hat{F}_{bh} - 2\sigma\Omega_b - k_b - \sigma}{1+2\sigma} & \text{if } \Omega_b \in (k_b - 1 - \sigma, k_b + \sigma], \\ \hat{F}_{bh} - \Omega_b - 1 & \text{otherwise.} \end{cases} \quad (54)$$

Because  $\Omega_b$  is uniformly distributed, we have that

$$\begin{aligned} \mathbb{E}\{G_\Sigma[\underline{\Sigma}(\Omega_b, z^*, z^*)]\} &= \\ &= \begin{cases} 0 & \text{if } z^* \leq -\omega - \sigma, \\ \frac{z^* + \sigma + \omega}{2\omega} \frac{z^* + \sigma}{1+2\sigma} - \frac{\sigma}{2\omega(1+2\sigma)} [(z^* + \sigma)^2 - \omega^2] & \text{if } z^* \in (-\omega - \sigma, \omega - \sigma], \\ \frac{z^* + \sigma}{1+2\sigma} & \text{if } z^* \in (\omega - \sigma, 1 - \omega + \sigma], \\ \frac{z^* - 1 + \omega - \sigma}{2\omega} + \frac{\omega - z^* + 1 + \sigma}{2\omega} \frac{z^* + \sigma}{1+2\sigma} + \frac{\sigma}{2\omega(1+2\sigma)} [\omega^2 - (z^* - 1 - \sigma)^2] & \text{if } z^* \in (1 - \omega + \sigma, 1 + \sigma + \omega], \\ 1 & \text{otherwise.} \end{cases} \end{aligned} \quad (55)$$

In combination with equation (53), this pins down the equilibrium threshold  $z^*$  with finite variances of both the idiosyncratic and systematic noise.

Finally, under  $\omega \rightarrow 0$  we find that the strategy played by all households in the unique Bayesian Nash equilibrium of the coordination game is

$$H_{bh}^* = \begin{cases} 1 & \text{if } \hat{F}_{bh} \geq \frac{j_b - \rho}{j_b - \rho + \theta} + \frac{j_b - \rho - \theta}{j_b - \rho + \theta} \sigma, \\ 0 & \text{otherwise.} \end{cases} \quad (56)$$

□

**Proposition 1.** *Consider vanishingly small idiosyncratic noise  $\sigma/\omega \rightarrow 0$  relative to systematic noise. Given fragility  $F_b$  and a funding spread  $j_b - \rho$ , the share  $H_b$  of households holding bank  $b$ 's deposits in equilibrium follows a Bernoulli distribution:*

$$\mathbb{P}[H_b] = \begin{cases} \kappa_b & \text{if } H_b = 1, \\ 1 - \kappa_b & \text{if } H_b = 0, \end{cases} \quad (10)$$

with

$$\kappa_b = \begin{cases} 1 & \text{if } j_b - \rho \geq \frac{F_b + \omega}{1 - F_b - \omega} \theta, \\ \frac{(j_b - \rho)(1 - F_b + \omega) - (F_b - \omega)\theta}{2\omega(j_b - \rho + \theta)} & \text{if } j_b - \rho \in \left[ \frac{F_b - \omega}{1 - F_b + \omega} \theta, \frac{F_b + \omega}{1 - F_b - \omega} \theta \right), \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

*Proof.* The probability that no one runs given  $F_b$  and  $j_b$  is given by the probability that

even the household drawing the highest signal is (weakly) below the run threshold:

$$\mathbb{P}[H_b = 1] = \mathbb{P}[F_b + \sigma + \Omega_b \leq F_b^*] = \mathbb{P}[\Omega_b \leq F_b^* - \sigma - F_b]. \quad (57)$$

This is the cdf of systematic noise  $\Omega_b$  evaluated at  $F_b^* - \sigma - F_b$ . Hence,  $\mathbb{P}[H_b = 1] = \varkappa_b$  where  $\varkappa_b$  is given by equation (11).

Now we prove that  $\mathbb{P}[H_b = 0] = 1 - \mathbb{P}[H_b = 1]$ . Partial runs with  $H_b \in (0, 1)$  imply that the household receiving the lowest signal holds deposits while the household receiving the highest signal does not. This event has probability  $\mathbb{P}[F_b - \sigma + \Omega_b \leq F_b^* \cap F_b + \sigma + \Omega_b > F_b^*]$ , which we can re-write as

$$\mathbb{P}\left\{\Omega_b \in [F_b^* - F_b - \sigma, F_b^* - F_b + \sigma]\right\} = G_\Omega(F_b^* - F_b + \sigma) - G_\Omega(F_b^* - F_b - \sigma) \leq \frac{\sigma}{\omega}, \quad (58)$$

where  $G_\Omega(\cdot)$  is the cdf of random variable  $\Omega_b$ . For  $\sigma/\omega \rightarrow 0$ , the proposition holds.  $\square$

**Proposition 2.** Consider  $\sigma/\omega \rightarrow 0$ . If  $\mathbb{P}[H_b = 1] = 1$  and  $F_b \leq 1 - 2(\sigma + \omega)$ , then household  $h$ 's belief about bank failure is almost surely correct with

$$\mathbb{P}_{bh}[\Phi_b = 1] = \mathbb{P}[\Phi_b = 1] = 0. \quad (59)$$

*Proof.* If a household has the belief that all households hold deposits with certainty  $\mathbb{P}_{bh}[H_b = 1] = 1$  and that a bank's fragility is not greater than one  $\mathbb{P}_{bh}[F_b \leq 1] = 1$ , then by the condition that determines bank failure (5), the household must also believe that the bank does not fail  $\mathbb{P}_{bh}[\Phi_b = 1] = 0$ .

A household is sure that all households hold, that is,  $H_b = 1$ , if  $\hat{F}_{bh} \leq F_b^* - 2\sigma$ . And a household is sure that a bank's fragility  $F_b$  is smaller than one if  $\hat{F}_{bh} \leq 1 - \sigma - \omega$ . This condition holds for all possible signals under  $F_b \leq 1 - 2(\sigma + \omega)$ . Hence, the sufficient condition for household  $h$  to believe for sure that  $\Phi_b = 0$  is simply given by  $\hat{F}_{bh} \leq F_b^* - 2\sigma$ . With this, we can write that for a given fragility  $F_b$ :

$$\mathbb{P}[\mathbb{P}_{bh}[\Phi_b = 1] = 0] \geq \mathbb{P}[\hat{F}_{bh} \leq F_b^* - 2\sigma] = \mathbb{P}[F_b + \Sigma_{bh} + \Omega_b \leq F_b^* - 2\sigma]. \quad (60)$$

Proposition 1 implies that a necessary and sufficient condition for  $\mathbb{P}[H_b = 1] = 1$  is  $F_b \leq F_b^* - \sigma - \omega$ . Substituting this into the equation above, we obtain

$$\mathbb{P}[\mathbb{P}_{bh}[\Phi_b = 1] = 0] \geq \mathbb{P}[\Sigma_{bh} + \Omega_b \leq \omega - \sigma]. \quad (61)$$

Under  $\sigma/\omega \leq 1$ , we can compute the right-hand side as  $1 - \sigma/(2\omega)$ . Hence, if  $\sigma/\omega \rightarrow 0$ , then the proposition holds.  $\square$

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