Online appendix to "The Macroeconomics of Liquidity in Financial Intermediation"

Davide Porcellacchia[a] Kevin D. Sheedy

European Central Bank London School of Economics

9 July 2026

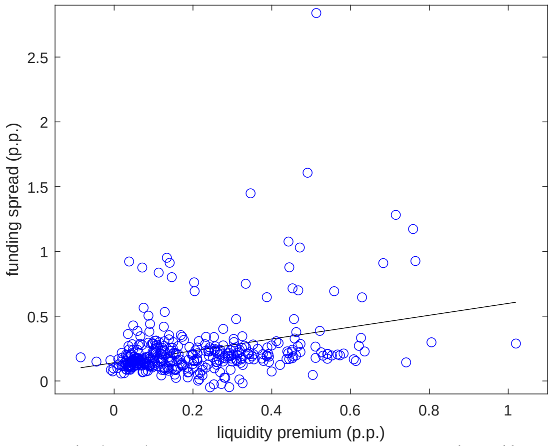
I. Data sources

We obtain the 3-month GC repo rate (mid-price from ticker 'USRGCGC ICUS Currncy') and the 3-month LIBOR from Bloomberg. Daily data on the quantity of outstanding Treasuries (series 'Debt held by the public' in dataset 'Debt to the Penny') and on the TGA closing balance (series 'Treasury General Account (TGA) Closing Balance' in dataset 'Daily Treasury Statement (DTS)') are available on the website Fiscaldata maintained by the US Treasury Department. From the website FRED maintained by the Federal Reserve Bank of St. Louis, we retrieve the 3-month T-bill rate (series 'DTB3'), the spread between Moody's seasoned Baa corporate bond yield and the 10-year Treasury rate (series 'BAA10Y'), the 10-year Treasury rate (series 'DGS10'), the VIX (series 'VIXCLS'), and the nominal broad US dollar index (series 'DTWEXBGS'). The closing values of the S&P 500 stock market index and the S&P 500 financials stock market index are downloaded from the website Yahoo! Finance.

II. Figures

[a]Contact: davide.porcellacchia@ecb.europa.eu

Figure 1: May 1991 – June 2023.



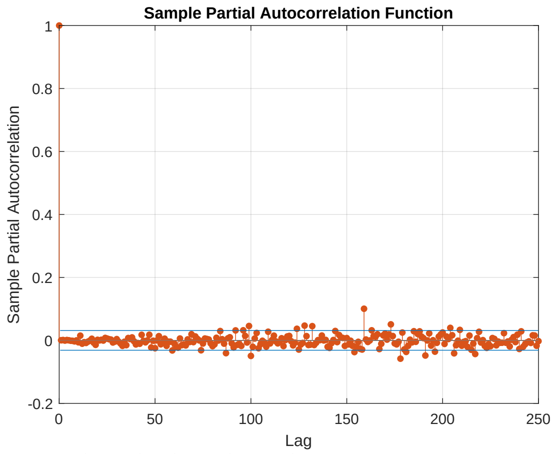
Note 1: Funding spread is 3-month (3M) LIBOR minus 3M general-collateral (GC) repo rate. Liquidity premium is 3M GC repo rate minus 3M T-bill rate.
Note 2: US daily data. Scatterplot of data at monthly frequency.

Table 1: OLS

Funding spread	OLS	OLS	OLS	OLS
Liquidity premium	0.75***	0.40***	-0.30***	-0.30***
	(0.06)	(0.04)	(0.06)	(0.06)
Lags	N	N	Y	Y
Time dummies	N	Y	N	Y
Linear trend	Y	Y	Y	Y
R-squared	23%	57%	99%	99%
Observations	4157	4157	4077	4077

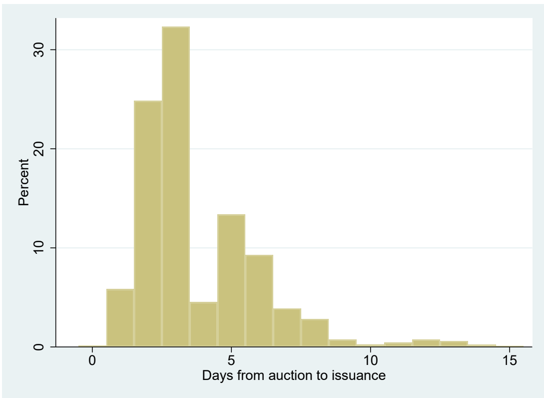
Note 1: Heteroscedasticity-consistent standard errors in parentheses.
Note 2: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

Figure 2: Partial autocorrelation function of funding-spread innovations.



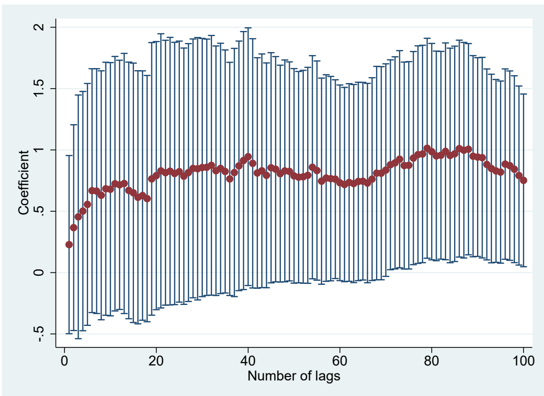
Note 1: The residuals are estimated in the benchmark IV regression.
Note 2: The blue lines are 95% confidence intervals for estimates of sample partial autocorrelation with a white-noise process.

Figure 3: Time from Treasury auction to issuance



Note 1: Data source is TreasuryDirect.gov.
Note 2: There are 5,320 observations of CUSIP-level Treasury bills, notes and bonds issued between January 2006 and June 2023.
Note 3: The instances of auction and issuance on the same day are 7, corresponding to 0.1% of observations.

Figure 4: Robustness to lag selection.



Note: The blue bar represents the 95% confidence interval for the coefficient of interest β estimated with IV.

III. Bank runs in the macroeconomic model

Since bank runs do not happen in equilibrium and households anticipate this, section 4 of the main text abstracts from bank runs in the analysis of household, firm, bank, and government behaviour. Nonetheless, the equilibrium of the coordination game depends on understanding what would happen if a run were to occur, and this appendix shows how this outcome is consistent with the macroeconomic model.

Banks. Each bank b makes choices at the competitive-markets stage that determine the variables A_{bt}, M_{bt}, D_{bt}, and N_{bt} on its balance sheet $A_{bt} + M_{bt} = D_{bt} + N_{bt}$, where $N_{bt} > 0$. If a positive fraction of households choose not to hold the bank's deposits ($H_{bt} < 1$) in the coordination game, its balance sheet must adapt. In what follows, the notation $\bar{\cdot}$ is used to denote the value of a variable at the coordination game or consumption stage of period t in the case of a run or partial run on one or more banks.

When faced with a run, a bank first disposes of its liquid assets M_{bt}, reducing them to $\bar{M}_{bt} = \max\{0, M_{bt} - (1 - H_{bt})D_{bt}\}$, which can be done without the bank suffering any loss. If this is insufficient to repay depositors, physical capital is liquidated down to $A_{bt} + (M_{bt} - \bar{M}_{bt}) - (1 - H_{bt})D_{bt}$. This is feasible given that net worth is positive and there is convertibility between physical capital and goods, with no adjustment cost suffered by the bank unless physical capital needs to be reduced below λA_{bt}.

However, liquidating bank b's physical capital below λA_{bt} (equivalent to $H_{bt} < F_{bt}$) causes the bank to fail ($\Phi_{bt} = 1$) and its net worth is lost, which acts as a capital adjustment cost. If the bank fails, any remaining depositors are repaid by liquidating further physical capital. In general, the amount of deposits still held by households at the end of the coordination game is $\bar{D}_{bt} = (1 - \Phi_{bt})H_{bt}D_{bt}$, where this formula applies both to surviving and failing banks. The bank's remaining physical capital is $\bar{A}_{bt} = A_{bt} - (D_{bt} - \bar{D}_{bt}) + (M_{bt} - \bar{M}_{bt}) - \Phi_{bt}N_{bt}$, which implies its post-run balance sheet is $\bar{A}_{bt} + \bar{M}_{bt} = \bar{D}_{bt} + \bar{N}_{bt}$ with net worth $\bar{N}_{bt} = (1 - \Phi_{bt})N_{bt}$. For a bank b that fails, capital adjustment costs $\Phi_{bt}N_{bt}$ wipe out all remaining assets ($\bar{A}_{bt} = 0$) and net worth ($\bar{N}_{bt} = 0$).

Runs that cause banks to dispose of illiquid assets affect the level of investment $\bar{I}_{bt}$ and the future capital stock $\bar{K}_{b,t+1} = X_{t+1}\bar{A}_{bt}$. Capital adjustment costs are a resource cost that are accounted for by including them as part of investment $\bar{I}_{bt} = \bar{A}_{bt} - (1 - \delta)K_{bt} + \Phi_{bt}N_{bt}$. A bank that fails at date t ceases to operate from date $t + 1$. Any depositors who were still holding at the point of bank failure incur recovery costs in the bankruptcy process, and these resource costs are paid at the beginning of period $t + 1$. Formally, these costs are treated like a capital adjustment cost and accounted for as part of investment expenditure, that is, for a bank b with $\Phi_{bt} = 1$, depositors' costs appear as $\bar{I}_{b,t+1} = \theta\Phi_{bt}H_{bt}D_{bt}$, which replaces the investment equation for banks that have previously failed.

Households. Based on past decisions and outcomes, household h begins period t with deposits including accrued interest $(1+j_{b,t-1})(1-\Phi_{b,t-1})D_{b,t-1}H_{bh,t-1}$ that were held at surviving banks b. The household pays at date t a cost $\theta\Phi_{b,t-1}D_{b,t-1}H_{bh,t-1}$ incurred in recovering funds on deposit at a failing bank b in period $t-1$.

Household h's choices at the competitive-markets stage of period t determine a level of spending power S_{ht} carried into the subsequent banking and consumption stages that is not directly held in government-issued liquid assets M_{ht}:

$$S_{ht} = w_t L_{ht} + \Pi_t + G_t - T_t + \int_0^1 \{(1+j_{b,t-1})(1-\Phi_{b,t-1}) - \theta\Phi_{b,t-1}\} D_{b,t-1}H_{bh,t-1}\,\mathrm{d}b \\ - (1+\rho_{t-1})B_{h,t-1} + B_{ht} + (1+i_{t-1})\bar{M}_{h,t-1} - M_{ht}, \quad \text{(OA.1)}$$

where $\bar{M}_{h,t-1}$ allows for holdings of liquid assets to change depending on outcomes at the banking stage. During the coordination game, households simultaneously make their deposit holding decisions $H_{bht} \in \{0,1\}$, which collectively determine any bank failures $\Phi_{bt} \in \{0,1\}$.

These decisions and outcomes for banks affect the consumption households can receive at the end of period t. Those choosing not to hold bank b's deposits can consume an extra amount D_{bt}. Those holding deposits at failing banks can recover them through the bankruptcy process by paying a per-unit cost θ at the beginning of the next period. After any withdrawals or bank failures, households may hold both goods and liquid assets, so these two markets are re-opened, though all other markets are closed at the final stage of period t.

In general, it is not automatic that liquid assets and goods would continue to exchange at the same one-for-one relative price from the first stage of period t, so here Λ_t specifies the market value of a unit of liquid assets in terms of goods at the final stage of the period. For liquid assets to have their defining characteristic, it must be shown that $\Lambda_t = 1$ in equilibrium, which will require the government to act in such a way that its liabilities M_t are indeed perfectly liquid.

Denoting household h's consumption for a general outcome of the coordination game by $\bar{C}_{ht}$:

$$\bar{C}_{ht} = S_{ht} - \bar{T}_t - \int_0^1 (1-\Phi_{bt}) H_{bht} D_{bt}\,\mathrm{d}b + \Lambda_t\left(M_{ht} - \bar{M}_{ht}\right), \quad \text{(OA.2)}$$

where $\bar{M}_{ht}$ is the amount of liquid assets held at the end of period and carried into period $t+1$, and $\bar{T}_t$ denotes a net lump-sum tax levied on all households after the coordination game.

Households' objective function is (19) and they are subject to (OA.1) and (OA.2) as constraints. The current-value Lagrangian multiplier on (OA.1) is μ_{ht}, and the outcome-contingent multiplier on (OA.2) is $\bar{\mu}_{ht}$. The first-order conditions with respect to B_{ht}, L_{ht}, S_{ht} (chosen at the competitive-markets stage at the beginning of the period), and $\bar{C}_{ht}$ (determined at the end of the period) are $\mu_{ht} = \beta(1+\rho_t)\mathbb{E}_{ht}[\mu_{h,t+1}]$, $w_t\mu_{ht} = \chi L_{ht}^{1/\psi}$,

$\mu_{ht} = \mathbb{E}_{ht}[\bar{\mu}_{ht}]$, and $\bar{\mu}_{ht} = \bar{C}_{ht}^{-1/\phi}$. Note that $\bar{\mu}_{ht}$ and $\bar{C}_{ht}$ are in general random variables because household h's information set does not perfectly predict outcomes in the coordination game and hence consumption.

Since there is a continuum of banks, the impact of any individual b on the constraint (OA.2) is small, so households act as if risk neutral in respect of their strategies in the coordination game. The coordination game is also played simultaneously across all banks, so current payoffs are valued using the expected Lagrangian multiplier $\mu_{ht} = \mathbb{E}_{ht}[\bar{\mu}_{ht}]$. Household h therefore discounts expected future payoffs using a discount factor $\beta\mathbb{E}_{ht}[\mu_{h,t+1}]/\mu_{ht}$, which is equal to $1/(1+\rho_t)$ using the B_{ht} first-order condition. The discount factor is thus common to all households, as supposed in the analysis of the coordination game, and is derived from the yield ρ_t on a risk-free but illiquid bond.

Government and market clearing. The government does not have access to any special production or storage technology allowing it directly to create an asset that is a liquid store of value. There is no reason why the value of a unit of M_t has to be the same at the first and final stages of period t, for example, if liquid assets held by banks were distributed to satisfy withdrawal demands. However, the government is able to adjust the supply of liquid assets to accommodate any changes in demand by varying its fiscal policy appropriately. The government's flow budget constraint at the final stage of period t (when only markets for goods and liquid assets are open) is $\bar{T}_t = \Lambda_t\left(M_t - \bar{M}_t\right)$, where $\bar{M}_t$ is the end-of-period supply of liquid bonds and $\bar{T}_t$ is a lump-sum tax levied on all households at the final stage.

The market for liquid bonds clears where $\bar{M}_{ht}$ and $\bar{M}_{bt}$ summed over households $h \in [0,1]$ and banks $b \in [0,1]$ equal the final supply of bonds $\bar{M}_t$. Assume the government sets the tax $\bar{T}_t$ equal to the quantity of liquid assets that are disbursed by banks faced with runs:

$$\bar{T}_t = \int_0^1 (M_{bt} - \bar{M}_{bt})\,\mathrm{d}b. \tag{OA.3}$$

Suppose that households do not want to hold liquid assets at either the beginning or end of period t, that is, $M_{ht} = 0$ and $\bar{M}_{ht} = 0$. Given (OA.3) and the government's budget constraint, the market for liquid assets at the end of period t clears only if $\Lambda_t = 1$. At the earlier, competitive-markets stage, market clearing requires $i_t \le \rho_t$, and under that condition, $M_{ht} = 0$ is optimal. Finally, $\bar{M}_{ht} = 0$ is also optimal if $(1+i_t)/(1+\rho_t) \le \bar{\mu}_{ht}/\mu_{ht}$. Budget constraints and the other market-clearing conditions imply goods-market equilibrium $\bar{C}_t + \bar{I}_t = Y_t$ holds, and from this, $\bar{M}_{ht} = 0$ is confirmed for a range of bank-run outcomes where $\bar{I}_t$ does not fall too far given that $\mu_{ht} = \mathbb{E}_t[\bar{\mu}_{ht}]$.

IV. Dividend policy and net worth

Banks' dividend policies are not the main focus of this paper, so the analysis is left to this appendix. Bank b's objective in choosing the path of dividends $\{\Pi_{bt}\}$ is to maximize the present discounted value of dividends $\Pi_{bt} + V_{bt}$, where V_{bt} from (24) is the present value of future dividends discounted using the representative-household stochastic discount factor P_{ts}. Assuming net worth is positive, the bank wants to satisfy the no-run condition (27) at all dates. The other constraints are its balance-sheet identity (26), the evolution of pre-dividend (and pre-bonus) net worth $E_{bt} = (1 + R_t)A_{b,t-1} + (1 + i_{t-1})M_{b,t-1} - (1 + j_{b,t-1})D_{b,t-1}$, post-dividend-and-bonus net worth $N_{bt} = E_{bt} - \Pi_{bt} - \Xi_{bt}$, and the amount of net worth $\Xi_{bt} = \max\{\gamma E_{bt}/(1+\gamma) - \Pi_{bt}, 0\}$ diverted as bonuses.

Start the analysis supposing that bank b's balance-sheet choices at date $t-1$ result in positive net worth E_{bt} with probability one. Note that if $\Pi_{bt} < \gamma E_{bt}/(1+\gamma)$ then $N_{bt} = E_{bt}/(1+\gamma)$, which is independent of Π_{bt}. It follows that current dividends Π_{bt} can be increased without affecting the path of future dividends, so banks always want to satisfy $\Pi_{bt} \geq \gamma E_{bt}/(1+\gamma)$, in which case $\Xi_{bt} = 0$ and post-dividend net worth $N_{bt} = E_{bt} - \Pi_{bt}$ satisfies (25). This can be expressed as the minimum dividend constraint $\Pi_{bt} \geq \gamma N_{bt}$ from (16).

Consider the present discounted value of future dividends V_{bt} in (24) starting from an initial level of net worth N_{bt} after setting the current dividend Π_{bt}. The Lagrangian for maximizing V_{bt} with respect to dividend policy $\{\Pi_{bs}\}_{s=t+1}^{\infty}$ subject to the sequence of minimum-dividend constraints from $t+1$ onwards is

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} P_{ts}\left\{\Pi_{bs} + \zeta_{bs}\left(\Pi_{bs} - \gamma N_{bs}\right)\right\}\right], \tag{OA.4}$$

where ζ_{bs} is the state-contingent Lagrangian multiplier on the date-s minimum dividend constraint of bank b, expressed in current-value terms, which means it is multiplied by the stochastic discount factor P_{ts} without loss of generality. Other constraints on the bank and optimal choices of other variables such as M_{bs} and D_{bs} are accounted for directly when evaluating (OA.4).

By adding and subtracting terms, the Lagrangian (OA.4) is equivalent to

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} P_{ts}\left\{(1+\zeta_{bs})\left(\Pi_{bs} + N_{bs}\right) - \left(1 + (1+\gamma)\zeta_{bs}\right)N_{bs}\right\}\right].$$

Defining variables W_{bt} and Ψ_{bt} as follows:

$$W_{bt} = \frac{\mathbb{E}_t\left[\Psi_{b,t+1}\left(N_{b,t+1} + \Pi_{b,t+1}\right)\right]}{\mathbb{E}_t\left[\Psi_{b,t+1}\right]}, \quad \text{where} \;\; \Psi_{b,t} = P_{t-1,t}(1+\zeta_{bt}), \tag{OA.5}$$

and noting that $P_{t,s+1} = P_{ts}P_{s,s+1}$ (see 24), the law of iterated expectations implies the

Lagrangian (OA.4) can be rewritten as follows in terms of these new variables:

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t}^{\infty} P_{ts}\Psi_{b,s+1}W_{bs} - \sum_{s=t+1}^{\infty} P_{ts}\left(1+(1+\gamma)\zeta_{bs}\right)N_{bs}\right]. \tag{OA.6}$$

Conjecturing that Ψ_{bt} is the same for all banks b, it follows from the definition (OA.5) that W_{bt} is the same as the static objective function (30) considered in section 5 with a common stochastic discount factor $\Psi_t = \Psi_{bt}$ used to calculated the risk-adjusted expected return $r_t = \mathbb{E}_t[\Psi_{t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t+1}]$.

The problem of maximizing W_{bt} conditional on net worth N_{bt}, taking the bank's dividend policy as given, has been studied in section 5. Optimizing over M_{bt} and D_{bt} subject to the balance sheet (26), the no-run condition (27), and accounting for the impact on future net worth (25), results in the maximized value of W_{bt} from (42) that is linear in initial net worth N_{bt} and with a common coefficient $1+(r_t-\lambda i_t)/(1-\lambda)$ for all banks. Substituting into (OA.6) yields a Lagrangian that depends only on the path of the bank's net worth:

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t}^{\infty} P_{ts}\Psi_{s+1}\left(1+\frac{r_s-\lambda i_s}{1-\lambda}\right)N_{bs} - \sum_{s=t+1}^{\infty} P_{ts}\left(1+(1+\gamma)\zeta_{bs}\right)N_{bs}\right], \tag{OA.7}$$

which also uses the conjecture $\Psi_{bt} = \Psi_t$. The bank then determines the path of net worth $\{N_{bs}\}_{s=t+1}^{\infty}$ by choosing dividends $\{\Pi_{bs}\}_{s=t+1}^{\infty}$. The first-order conditions are $\partial\Upsilon_{bt}/\partial N_{bs} = 0$ for $s = t+1, t+2, \ldots$ in all states of the world, and the associated Kuhn-Tucker conditions are $\zeta_{bs} \geq 0$, $\Pi_{bs} - \gamma N_{bs} \geq 0$, and $\zeta_{bs}(\Pi_{bs} - \gamma N_{bs}) = 0$. From (OA.7), the first-order conditions are

$$1+(1+\gamma)\zeta_{bs} = \left(1+\frac{r_s-\lambda i_s}{1-\lambda}\right)\mathbb{E}_s[\Psi_{s+1}]. \tag{OA.8}$$

This shows that the Lagrangian multiplier ζ_{bs} is the same for all banks if Ψ_{s+1} and r_s are common to all banks. Referring to (OA.5), that earlier conjecture is confirmed when the Lagrangian multipliers are the same for all b, the common value being denoted by ζ_t in what follows for a date-t constraint.

Using the Kuhn-Tucker conditions satisfied by the solution of the constrained maximization problem, the Lagrangian Υ_{bt} in (OA.4) equals the present-value of future dividends V_{bt} from (24). By substituting the first-order conditions (OA.8) for $s = t+1, t+2, \ldots$ into the expression for Υ_{bt} in (OA.7), only one non-zero term in N_{bt} remains. This establishes the present-discount value of future dividends V_{bt} is linear in net worth N_{bt}:

$$V_{bt} = v_t N_{bt}, \quad \text{where } v_t = \left(1+\frac{r_t-\lambda i_t}{1-\lambda}\right)\mathbb{E}_t[\Psi_{t+1}] \text{ and } \Psi_{t+1} = P_{t,t+1}(1+\zeta_{t+1}). \tag{OA.9}$$

The coefficient v_t of net worth is the market-to-book ratio: the market value of a bank's future dividends divided by its ex-dividend net worth. This ratio is common to all banks. Comparison of v_t from (OA.9) to the first-order conditions (OA.8) shows that $v_s = 1+(1+\gamma)\zeta_s$ for $s \geq t+1$. From the Kuhn-Tucker conditions, the market-to-book ratio satisfies $v_s \geq 1$, and the date-s minimum-dividend constraint binds if $v_s > 1$. Considering the choice of the date-t dividend Π_{bt}, bank b wants to maximize

$\Pi_{bt} + V_{bt} = (\Pi_{bt} + N_{bt}) + (v_t - 1)N_{bt}$, which uses (OA.9). Since $\Pi_{bt} + N_{bt}$ depends only on past decisions (see 25), the minimum dividend constraint binds if $v_t > 1$.

It has been supposed that banks' choices do not lead to equity $N_{b,t+1}$ becoming non-positive for any realization of $Q_{b,t+1}$ in (43). Recall that recapitalization by the investment fund costs $1+\xi$ units of net worth from other banks for each unit of capital injected into an insolvent bank. First note that since $V_{bt} = v_t N_{bt}$ with $v_t \geq 1$ for solvent banks, this recapitalization reduces the present value of dividends the investment fund is able to distribute to households. Second, the leverage and liquidity choices of individual banks are not uniquely pinned down by the optimality condition that equalizes fragility. It follows that as long as aggregate bank net worth does not become negative in any state of the world, individual banks can choose leverage to ensure that net worth remains positive with probability one without having to take actions that reduce their present discounted value of dividends.

V. Steady state

In this appendix, we analyse the long-run dynamics of the model by studying its steady state. The model's steady state is a constant sequence for prices and quantities that satisfies the model's equilibrium conditions.

We look for a steady state with a strictly positive liquidity premium $\rho - i > 0$ and bank net worth $N > 0$. Combining equations (38) and (44), we obtain

$$q - \rho = \theta + 2\sqrt{\theta(\rho - i)} > 0. \qquad \text{(OA.10)}$$

Evaluating the formula for the banks' market-to-book ratio (45) in steady state, we obtain

$$v = \frac{\gamma(1+q)}{(1+\gamma)(1+\rho) - (1+q)} > 1, \qquad \text{(OA.11)}$$

which implies that the minimum dividend constraint is binding in steady state so that $\Pi = \gamma N$. Together with the law of motion for banks' net worth in (46), a binding minimum dividend constraint implies that in a steady state with $N > 0$:

$$q = \gamma. \qquad \text{(OA.12)}$$

The upper limit on M identified below rules out $N = 0$ and $q < \gamma$ in a steady state with a strictly positive liquidity premium.

First, we notice from (OA.10) that the parametric restriction $\gamma > \rho + \theta$ is necessary for (OA.12) to be sustained with a strictly positive liquidity premium. If this is violated, then net worth grows up to the point where there is no fragility and the liquidity premium is zero. Under this restriction, we pin down the steady-state liquidity premium as

$$\rho - i = \frac{[\gamma - (\rho + \theta)]^2}{4\theta}. \qquad \text{(OA.13)}$$

This liquidity premium creates the right level of returns on bank net worth so that bank net worth is stable. Interestingly, it is independent of policy. Also, (38) implies

$$r - i = 4(1-\lambda)\left(\frac{1}{2}\sqrt{\theta} + \frac{1}{2}\sqrt{\rho - i}\right)^2. \tag{OA.14}$$

Moreover, we can determine the steady-state balance-sheet structure of banks with equations (26), (27) and (37) as

$$N = \left[1 - \lambda - \frac{\gamma - (\rho + \theta)}{2\theta}\left(\lambda + \frac{M}{K}\right)\right]K. \tag{OA.15}$$

To have positive net worth in steady state, we need to restrict the equity friction with

$$\gamma \leq \rho + \theta\frac{2-\lambda}{\lambda}, \tag{OA.16}$$

and policy with

$$M < \left[\frac{2\theta(1-\lambda)}{\gamma - (\rho + \theta)} - \lambda\right]K. \tag{OA.17}$$

An excessively strong equity friction makes it impossible to sustain positive net worth in steady state even with no liquidity. Excessively large liquid-asset supply rules out a fragile steady state with positive liquidity premium for any positive level of net worth.

The key finding that in the long run liquidity policy has no effect on the liquidity premium, and thus fragility, is due to the endogenous structure of banks' balance sheet. Increases in liquid-asset supply crowd out bank net worth in the long run to the point where fragility is unchanged.

As is standard in a real business cycle model, the steady-state risk-free rate is pinned down by the Euler equation in (21) as $\rho = (1-\beta)/\beta$ and the steady-state level of capital is the unique strictly-positive solution of the system of equations given by

$$(1-\alpha)K^{\alpha\left(1+\frac{1}{\phi}\right)} = L^{\alpha+\frac{1}{\psi}}\left(ZL^{1-\alpha} - \delta\right)^{-\frac{1}{\phi}}, \tag{OA.18}$$

and

$$K = \left(\frac{\alpha}{r-\delta}\right)^{\frac{1}{1-\alpha}} L. \tag{OA.19}$$

VI. Solving the full macroeconomic model

The full macroeconomic model is represented by the following set of equations:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{OA.20}$$

$$C_t + I_t = Y_t, \tag{OA.21}$$

$$(1-\alpha)\frac{Y_t}{L_t} = w_t, \tag{OA.22}$$

$$C_t^{\frac{1}{\phi}} L_t^{\frac{1}{\psi}} = w_t, \tag{OA.23}$$

$$K_t = X_t A_{t-1}, \tag{OA.24}$$

$$A_t = (1-\delta)K_t + I_t, \tag{OA.25}$$

$$P_t = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\frac{1}{\phi}}, \tag{OA.26}$$

$$\frac{1}{1+\rho_t} = \mathbb{E}_t P_{t+1}, \tag{OA.27}$$

$$R_t = \left(\alpha\frac{Y_t}{K_t} + 1 - \delta\right)\frac{K_t}{A_{t-1}} - 1, \tag{OA.28}$$

$$r_t = \frac{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})R_{t+1}]}{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})}], \tag{OA.29}$$

$$A_t + M_t = D_t + N_t, \tag{OA.30}$$

$$N_t = \frac{1+Q_t}{1+\gamma}N_{t-1}, \tag{OA.31}$$

$$F_t = 1 - \lambda - \frac{\lambda N_t + (1-\lambda)M_t}{D_t}, \tag{OA.32}$$

$$Q_t = q_{t-1} + (R_t - r_{t-1})\frac{A_{t-1}}{N_{t-1}}, \tag{OA.33}$$

$$r_t = (1-\lambda)q_t + \lambda i_t, \tag{OA.34}$$

$$r_t - i_t = (1-\lambda)\left(\sqrt{\theta} + \sqrt{\rho_t - i_t}\right)^2, \tag{OA.35}$$

$$j_t - \rho_t = \sqrt{\theta}\sqrt{\rho_t - i_t}, \tag{OA.36}$$

$$\rho_t - i_t = \theta\frac{F_t^2}{(1-F_t)^2}, \tag{OA.37}$$

$$V_t = \mathbb{E}_t[P_{t+1}(V_{t+1} + \Pi_{t+1})], \tag{OA.38}$$

$$\Pi_t = \gamma N_t. \tag{OA.39}$$

The variables Z_t and X_t denote exogenous levels of TFP and capital quality. It is necessary to add an equation describing how the supply of liquidity M_t is determined, which could be exogenous or endogenous. In what follows, $\tilde{}$ denotes the deviation of a variable from its steady-state value: a simple deviation for variables already measured as percentages, and log deviations for all other variables.

The production function (OA.20) in log deviations is:

$$\tilde{Y}_t = \tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{L}_t. \tag{OA.40}$$

Letting $\kappa = K/Y$ denote the steady-state capital-output ratio, since $I = \delta K$, the log linearization of the aggregate demand equation (OA.21) is:

$$(1-\delta\kappa)\tilde{C}_t + \delta\kappa\tilde{I}_t = \tilde{Y}_t. \tag{OA.41}$$

Labour demand (OA.22) and supply (OA.23) in log deviations are:

$$\tilde{Y}_t - \tilde{L}_t = \tilde{w}_t, \tag{OA.42}$$

$$\frac{1}{\phi}\tilde{C}_t + \frac{1}{\psi}\tilde{L}_t = \tilde{w}_t. \tag{OA.43}$$

Since $X = 1$, the supply of capital to firms (OA.24) and capital investment (OA.25) have the following log-linear forms:

$$\tilde{K}_t = \tilde{X}_t + \tilde{A}_{t-1}, \tag{OA.44}$$

$$\tilde{A}_t = (1-\delta)\tilde{K}_t + \delta\tilde{I}_t. \tag{OA.45}$$

Using $P = \beta$ and $\rho = (1-\beta)/\beta$, the stochastic discount factor P_t from (OA.26) and the implied risk-free rate (OA.27) in terms of deviations from steady state are:

$$\tilde{P}_t = -\frac{1}{\phi}(\tilde{C}_t - \tilde{C}_{t-1}), \tag{OA.46}$$

$$\tilde{\rho}_t = -(1+\rho)\mathbb{E}_t\tilde{P}_{t+1}. \tag{OA.47}$$

Using $r = R = (\alpha/\kappa) - \delta$, the equations (OA.28) and (OA.29) for the ex-post and risk-adjusted expected returns on physical capital have the following approximate forms:

$$\tilde{R}_t = (r+\delta)(\tilde{Y}_t - \tilde{K}_t) + (1+r)(\tilde{K}_t - \tilde{A}_{t-1}), \tag{OA.48}$$

$$\tilde{r}_t = \mathbb{E}_t\tilde{R}_{t+1}. \tag{OA.49}$$

In terms of $n = N/(A+M)$ and $m = M/(A+M)$, the bank balance sheet (OA.30) and the accumulation of net worth (OA.31) equations can be approximated as follows (noting $Q = \gamma$):

$$(1-m)\tilde{A}_t + m\tilde{M}_t = (1-n)\tilde{D}_t + n\tilde{N}_t, \tag{OA.50}$$

$$\tilde{N}_t = \tilde{N}_{t-1} + \frac{\tilde{Q}_t}{1+\gamma}. \tag{OA.51}$$

The approximation of the equation for bank fragility (OA.32) is:

$$\tilde{F}_t = \frac{(\lambda n + (1-\lambda)m)(1-m)}{(1-n)^2}\tilde{A}_t - \frac{(\lambda + (1-\lambda)m)n}{(1-n)^2}\tilde{N}_t - \frac{((1-\lambda)(1-m)-n)m}{(1-n)^2}\tilde{M}_t. \tag{OA.52}$$

Equations (OA.33), (OA.34) and (OA.35) for the returns on bank assets and liabilities become:

$$\tilde{Q}_t = \tilde{q}_{t-1} + \frac{(1-m)}{n}(\tilde{R}_t - \tilde{r}_t), \tag{OA.53}$$

$$\tilde{r}_t = (1-\lambda)\tilde{q}_t + \lambda\tilde{i}_t, \tag{OA.54}$$

$$\tilde{r}_t - \tilde{i}_t = (1-\lambda)\left(1 + \frac{\sqrt{\theta}}{\sqrt{\rho - i}}\right)(\tilde{\rho}_t - \tilde{i}_t). \tag{OA.55}$$

Equations (OA.36) and (OA.37) linking bank fragility, funding costs, and the liquidity premium have the following approximations:

$$\tilde{j}_t - \tilde{\rho}_t = \frac{1}{2}\frac{\sqrt{\theta}}{\sqrt{\rho - i}}(\tilde{\rho}_t - \tilde{i}_t), \tag{OA.56}$$

$$\tilde{\rho}_t - \tilde{i}_t = 2\frac{\sqrt{\rho - i}}{\sqrt{\theta}}\left(\sqrt{\theta} + \sqrt{\rho - i}\right)^2 \tilde{F}_t. \tag{OA.57}$$

The log linearization of the stock-market value equation (OA.38) is:

$$\tilde{V}_t = \frac{1}{1+\rho}\mathbb{E}_t \tilde{V}_{t+1} + \frac{\rho}{1+\rho}\mathbb{E}_t \tilde{N}_{t+1} - \frac{1}{1+\rho}\tilde{\rho}_t. \tag{OA.58}$$

Together with these main equations, there are auxiliary equations defining other variables. The funding spread is $\tilde{j}_t - \tilde{\rho}_t$, the liquidity premium $\tilde{\rho}_t - \tilde{i}_t$, the credit spread $\tilde{r}_t - \tilde{i}_t$, the total size of banks' balance sheets $(1-m)\tilde{A}_t + m\tilde{M}_t$, the capitalization ratio $\tilde{n}_t = n(1-n)(\tilde{N}_t - \tilde{D}_t)$, the liquidity ratio $\tilde{m}_t = m(1-m)(\tilde{M}_t - \tilde{A}_t)$, and the market-to-book value ratio $\tilde{v}_t = \tilde{V}_t - \tilde{N}_t$.